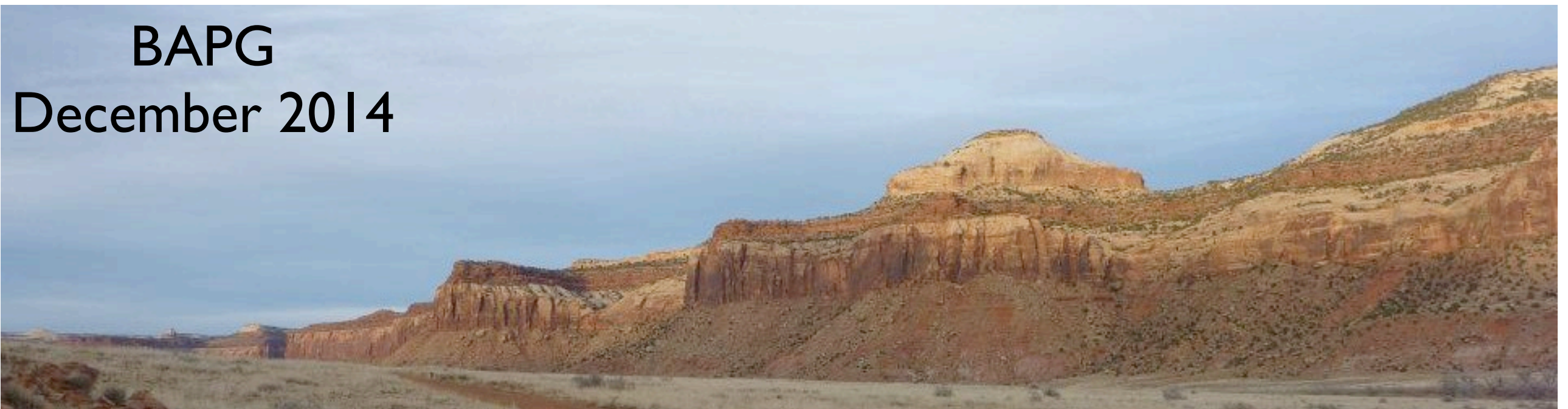
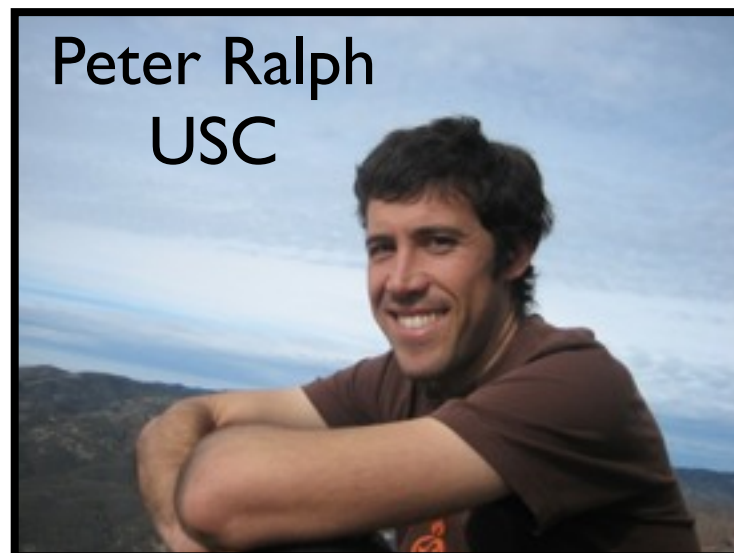


The Geography of Genetic Structure and Admixture



Gideon Bradburd
UC Davis
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The Geography of Genetic Structure and Admixture

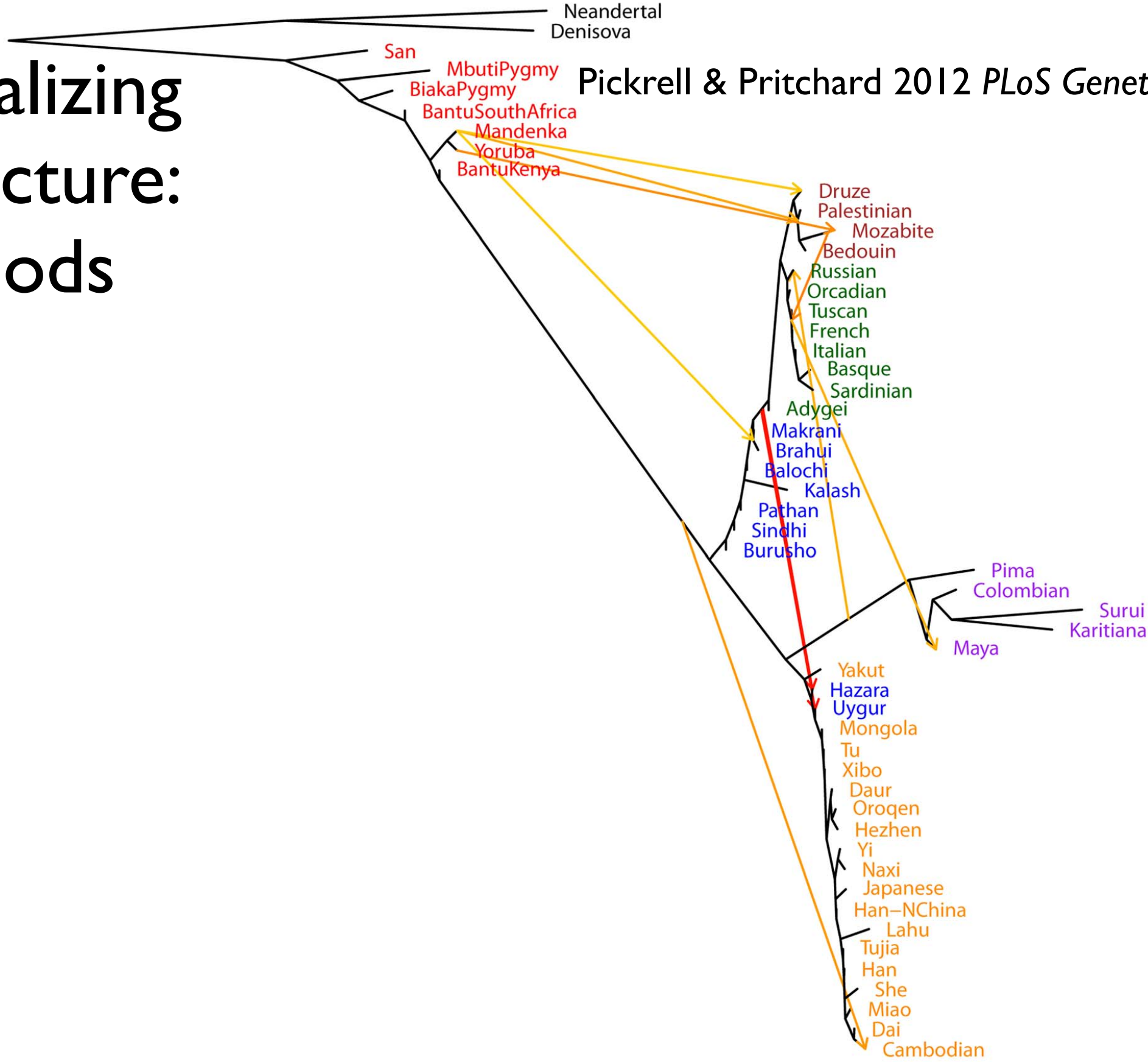


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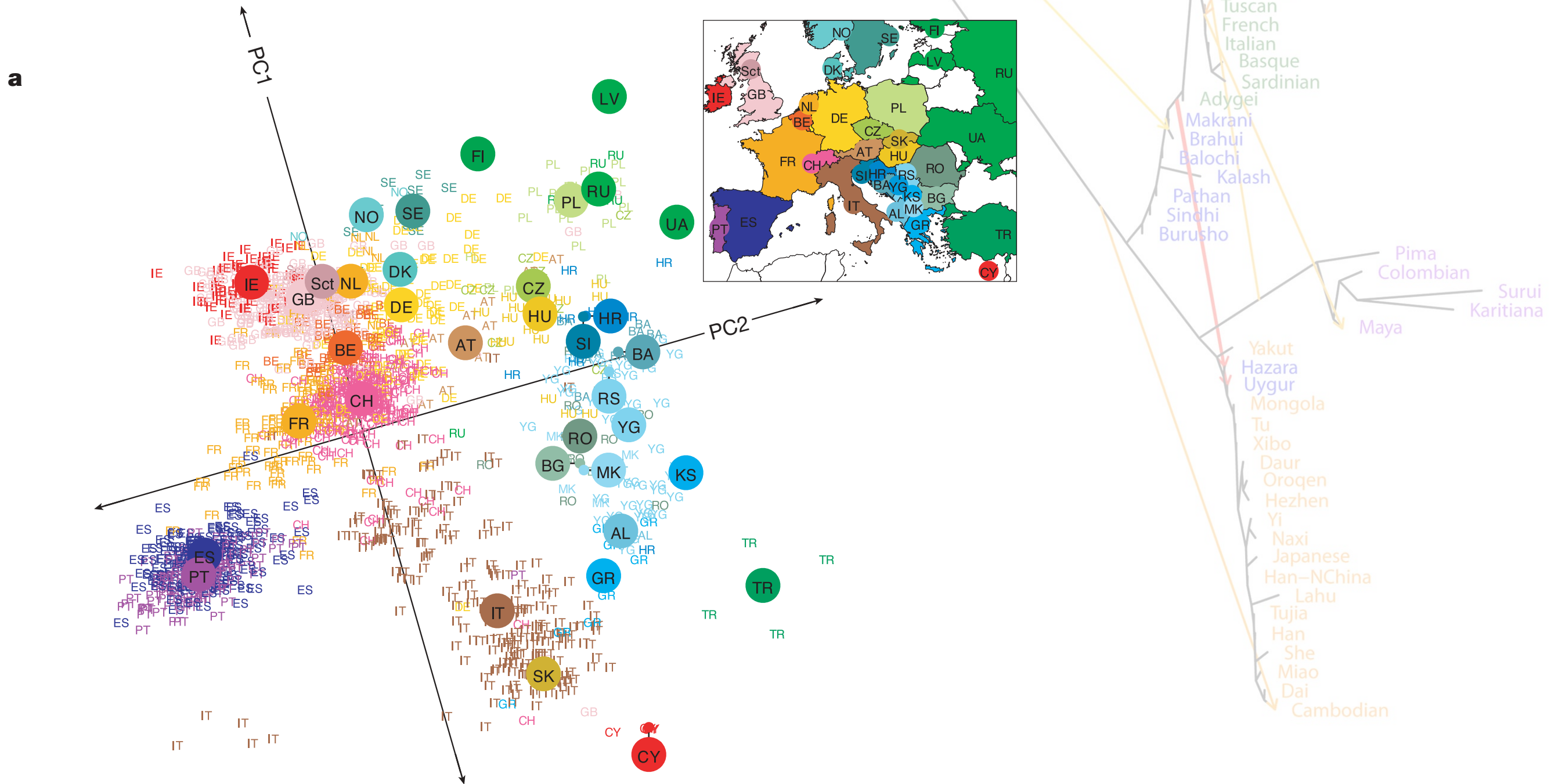


inferring & visualizing population structure: current methods

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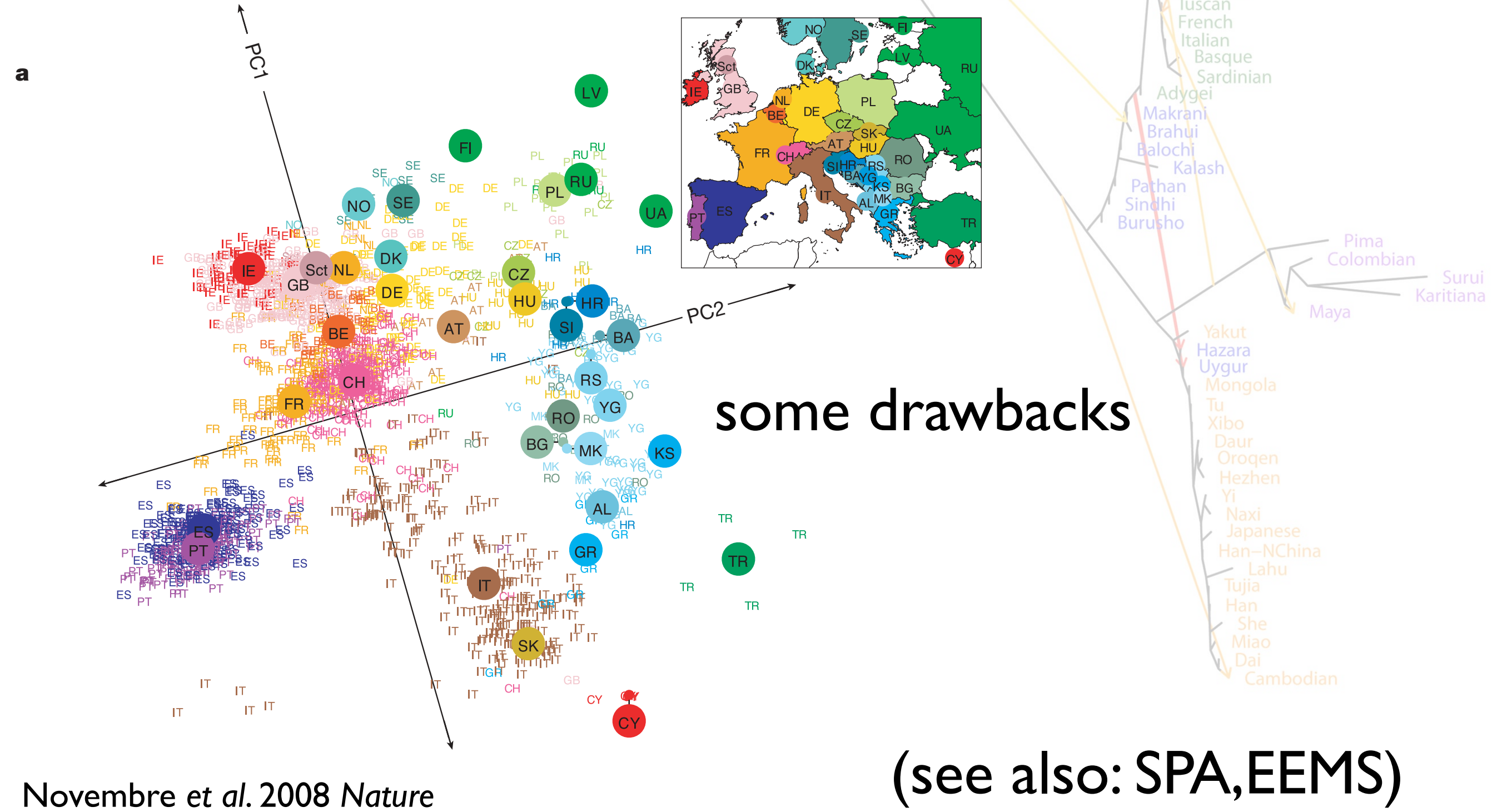
inferring & visualizing population structure: current methods



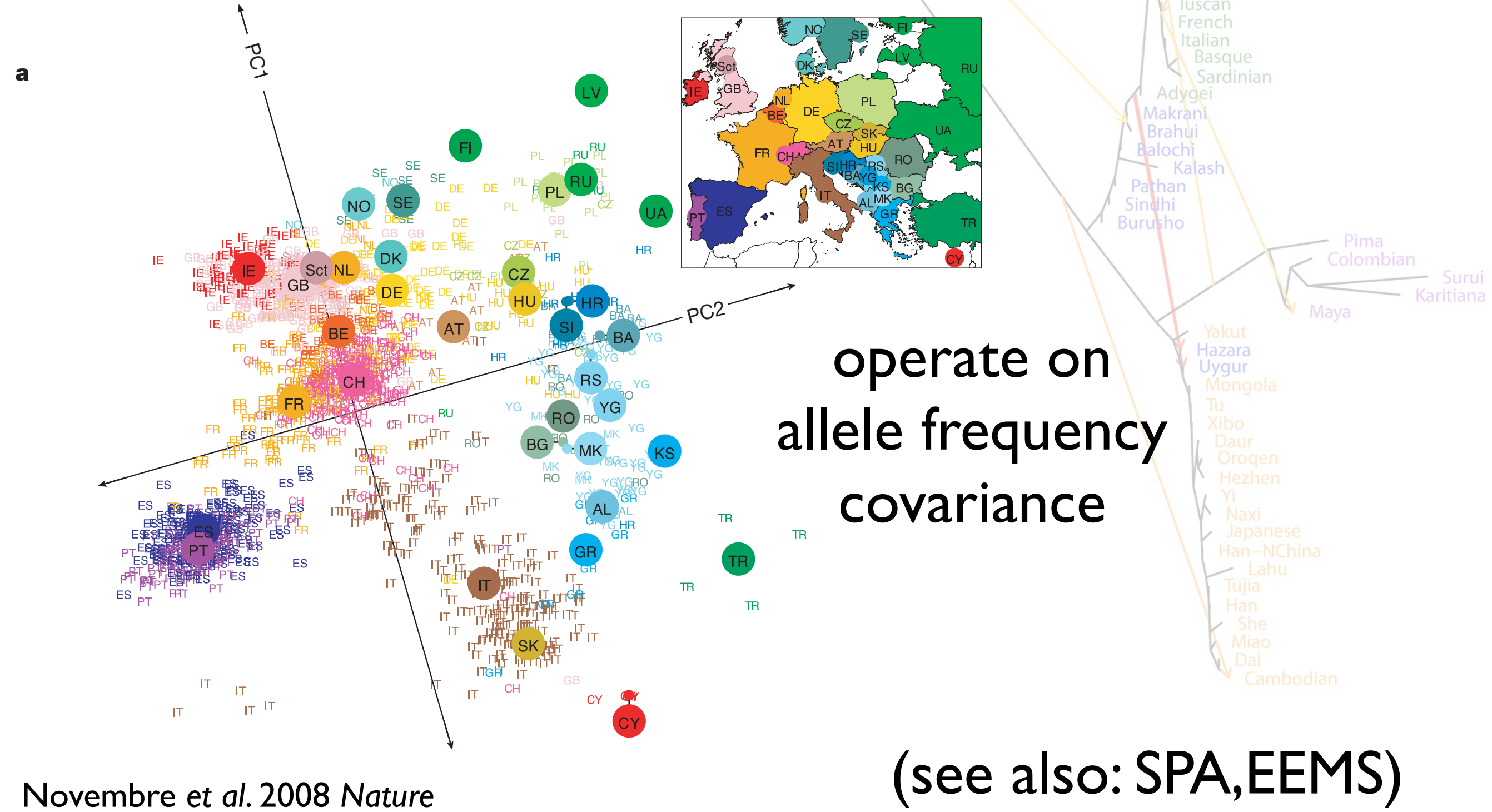
Novembre et al. 2008 *Nature*

(see also: SPA, EEMS)

inferring & visualizing population structure: current methods



inferring & visualizing population structure: current methods



SpaceMix Data:

$\hat{f}_{\ell,k}$ = sample frequency at locus ℓ in population k

$X_{\ell,k} = \frac{\hat{f}_{\ell,k} - \epsilon_{\ell}}{\sqrt{\epsilon_{\ell}(1 - \epsilon_{\ell})}} =$ standardized sample frequency
at locus ℓ in population k

$\hat{\Omega} = \frac{1}{L} X X^T =$ standardized sample covariance
across all L loci

SpaceMix Model: Isolation by Distance

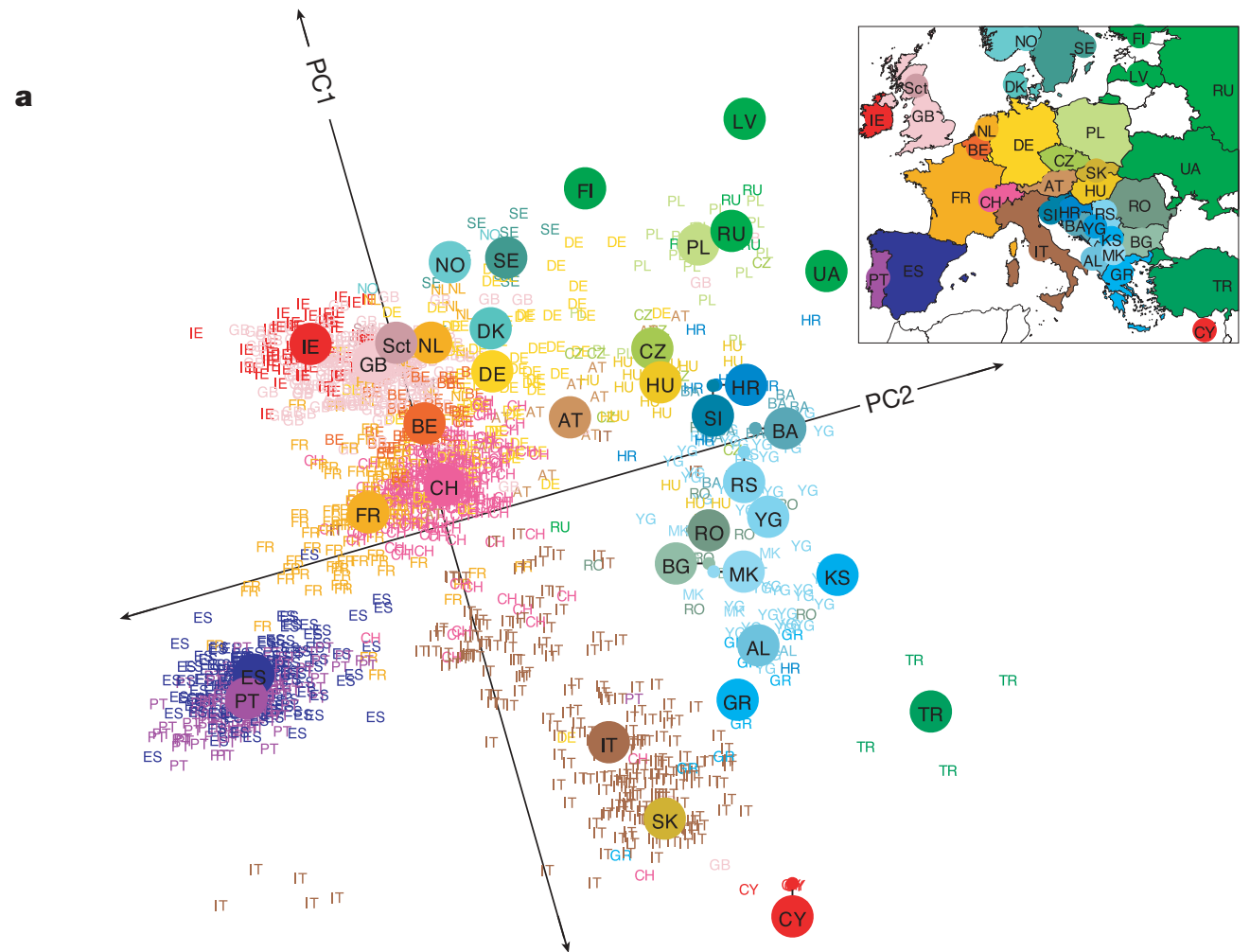
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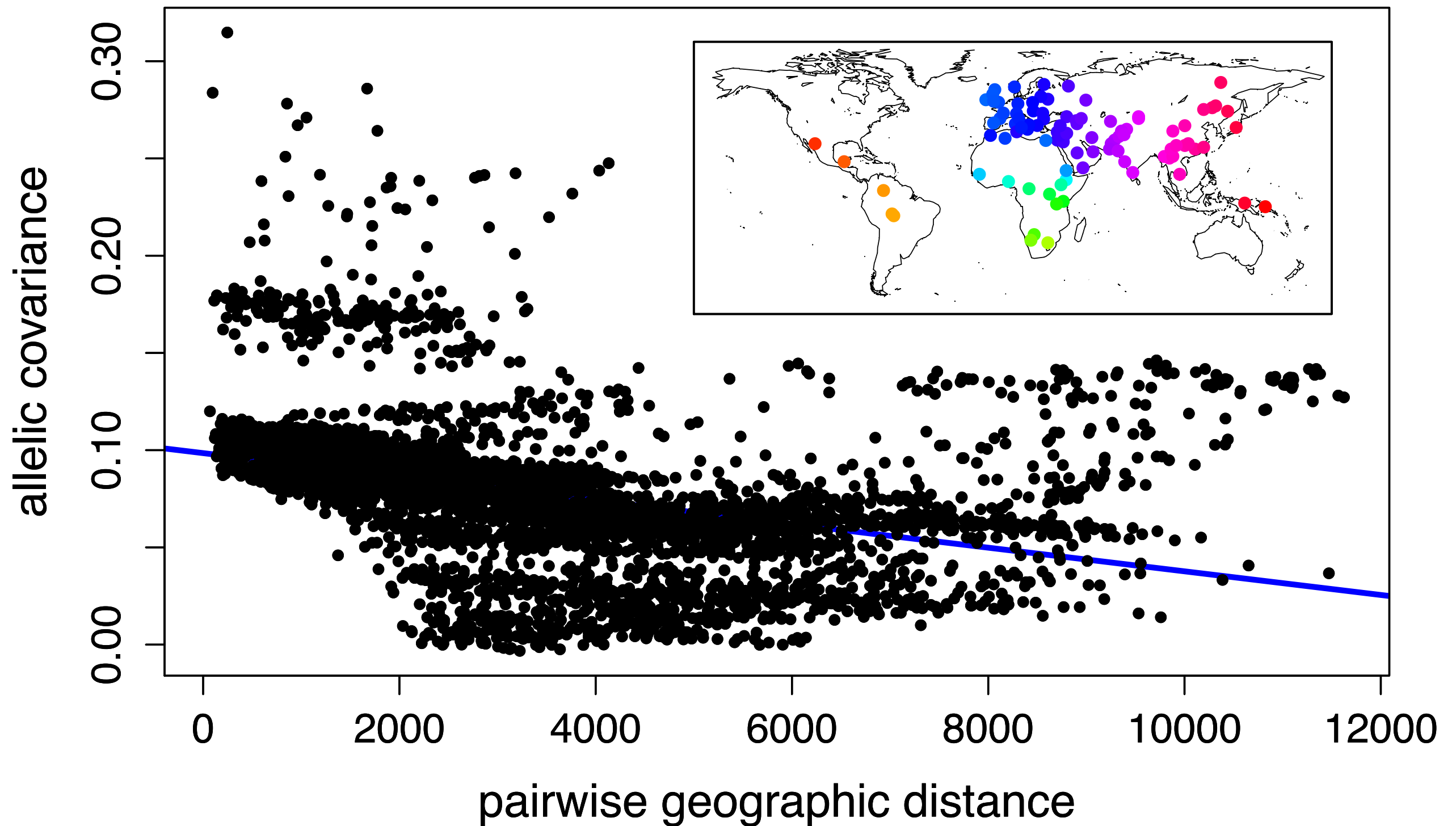
Isolation by Distance

Wright 1943



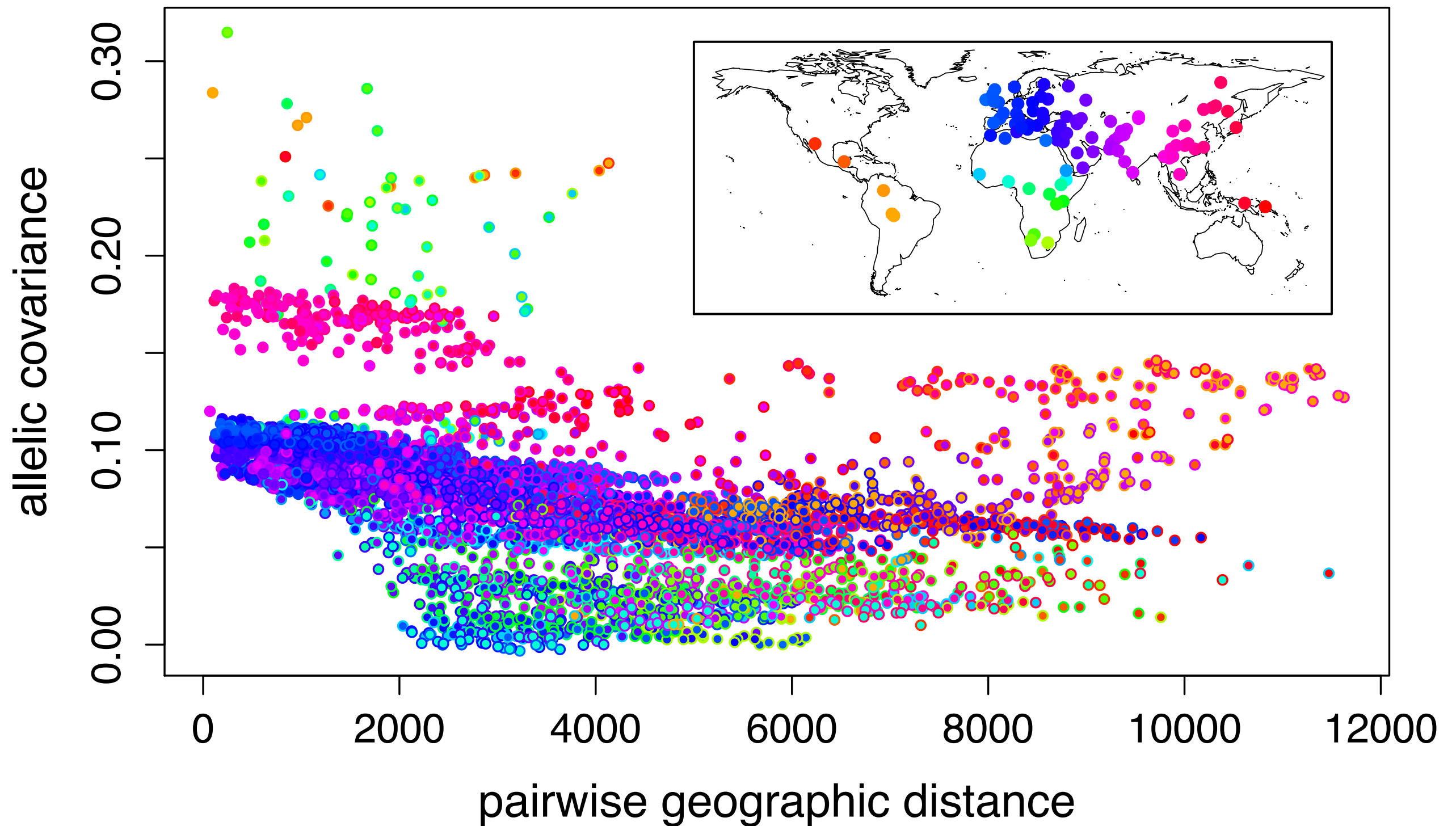
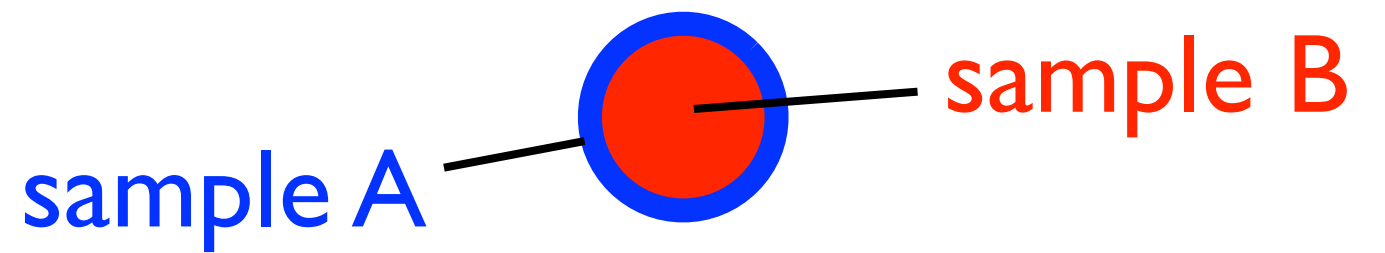
“Everything is related to everything else,
but near things are more related than distant things”
-Waldo Tobler 1970

Isolation by Distance: empirical example

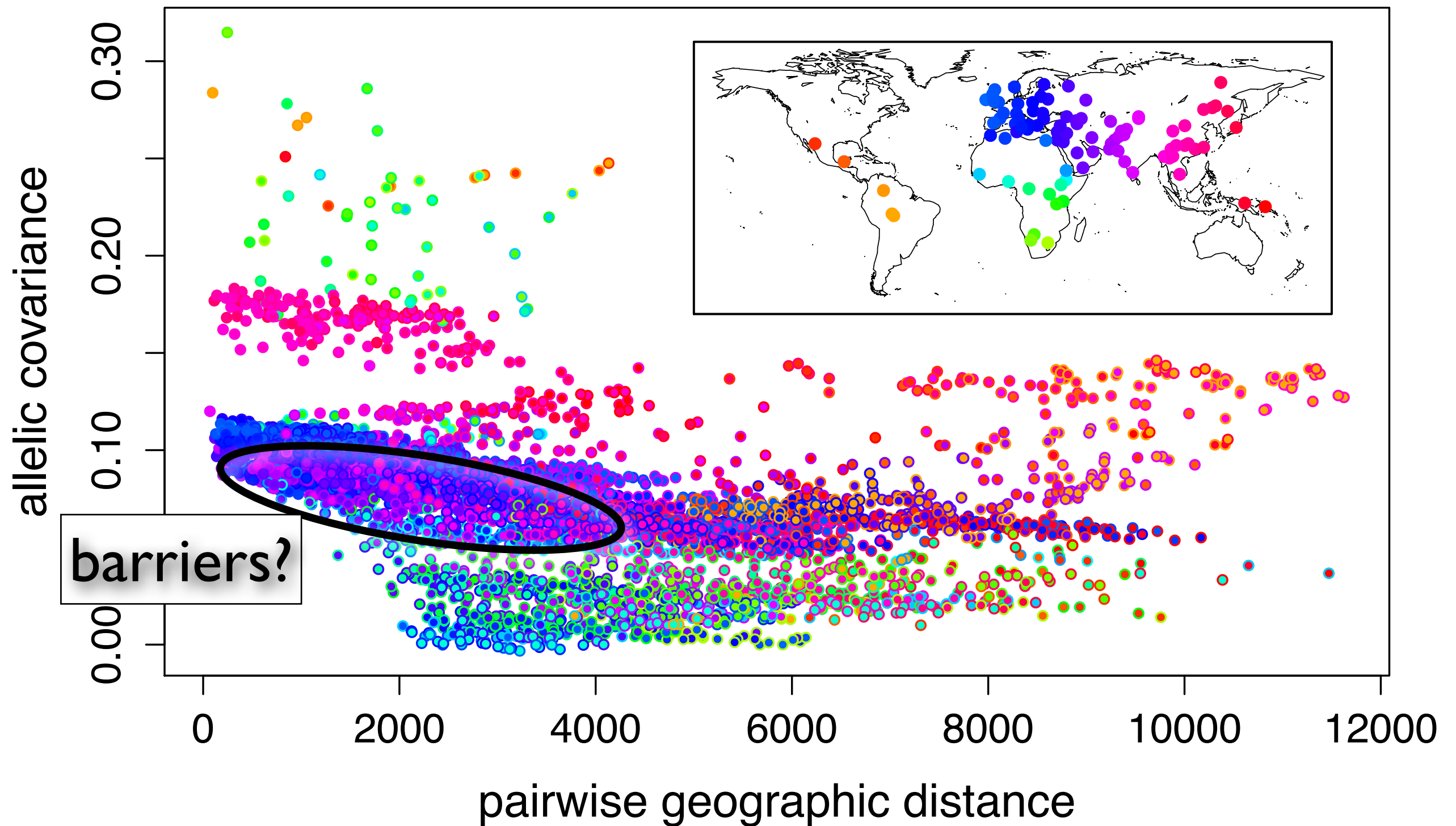


Hellenthal *et al.* 2014 *Science*

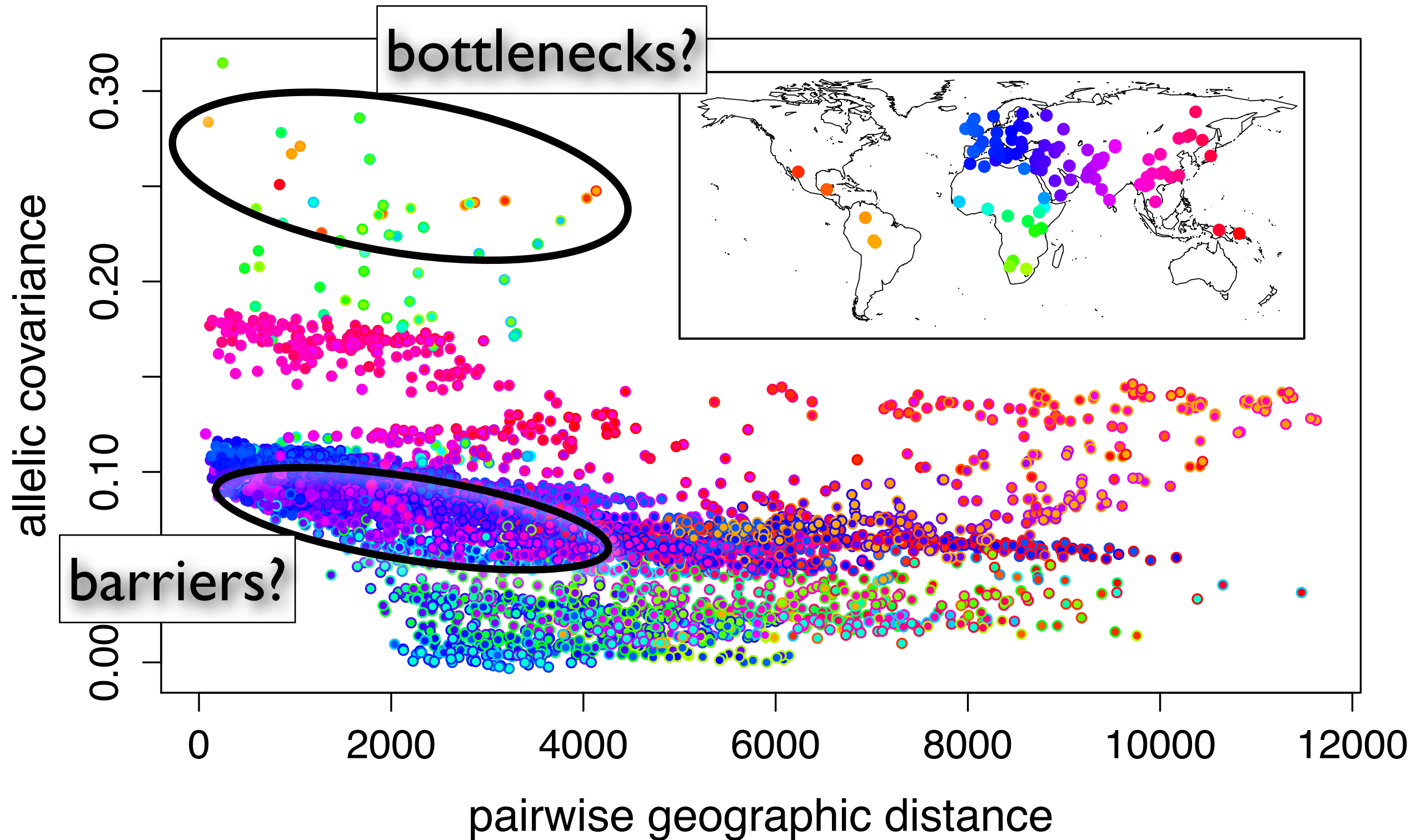
Isolation by Distance: empirical example



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Isolation by Distance: empirical example



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$$\Omega_{i,j} = \frac{1}{\alpha_0} \exp \left(-(\alpha_1 D_{i,j})^{\alpha_2} \right)$$

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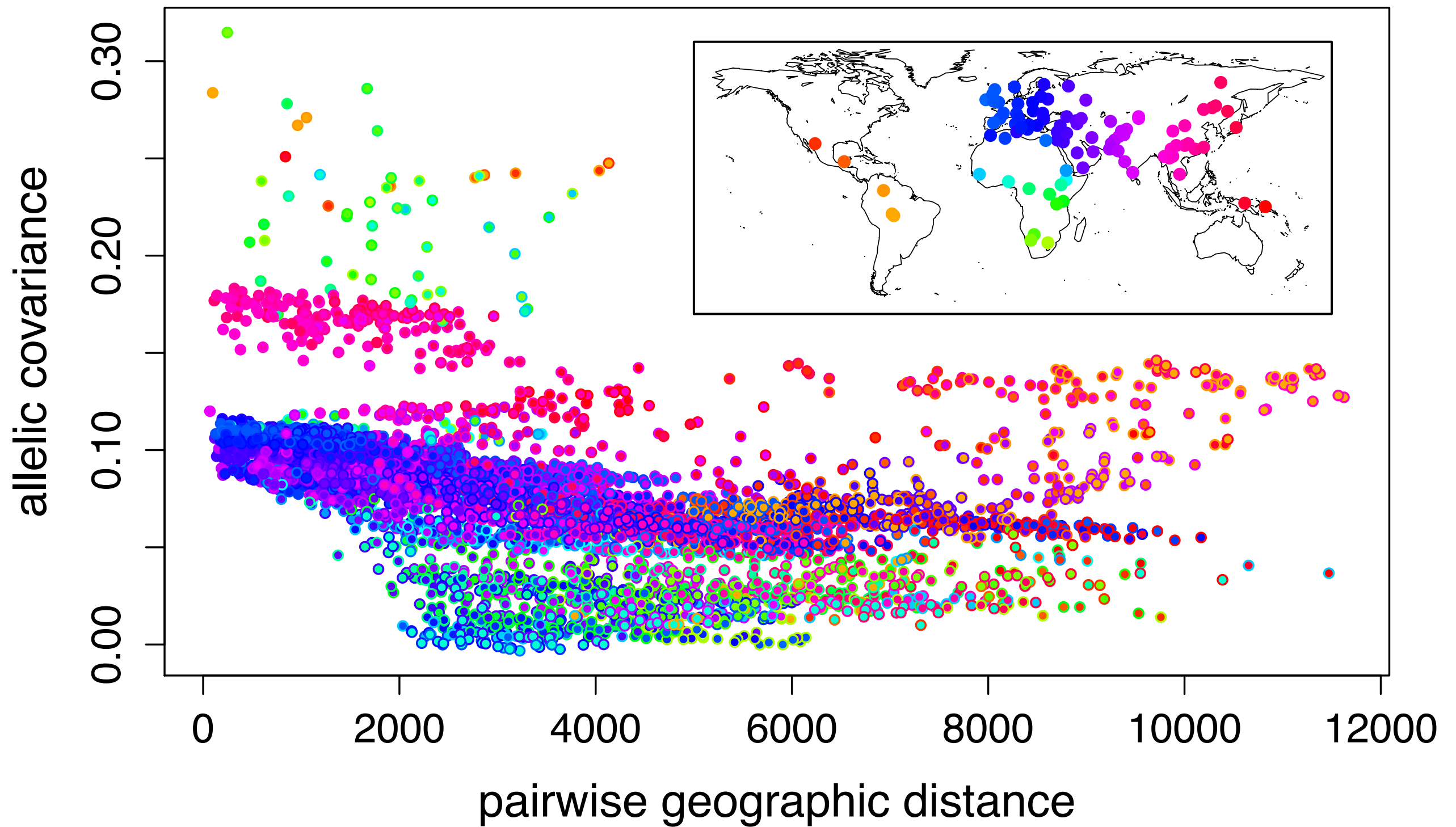
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$$\Omega = \text{function}(\alpha_0, \alpha_1, \alpha_2)$$

$$P(\Omega \mid \hat{\Omega}) \propto P(\hat{\Omega} \mid \Omega, L) P(\vec{\alpha})$$

Isolation by Distance



$X \sim$ spatial Gaussian process

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$X \sim$ spatial Gaussian process

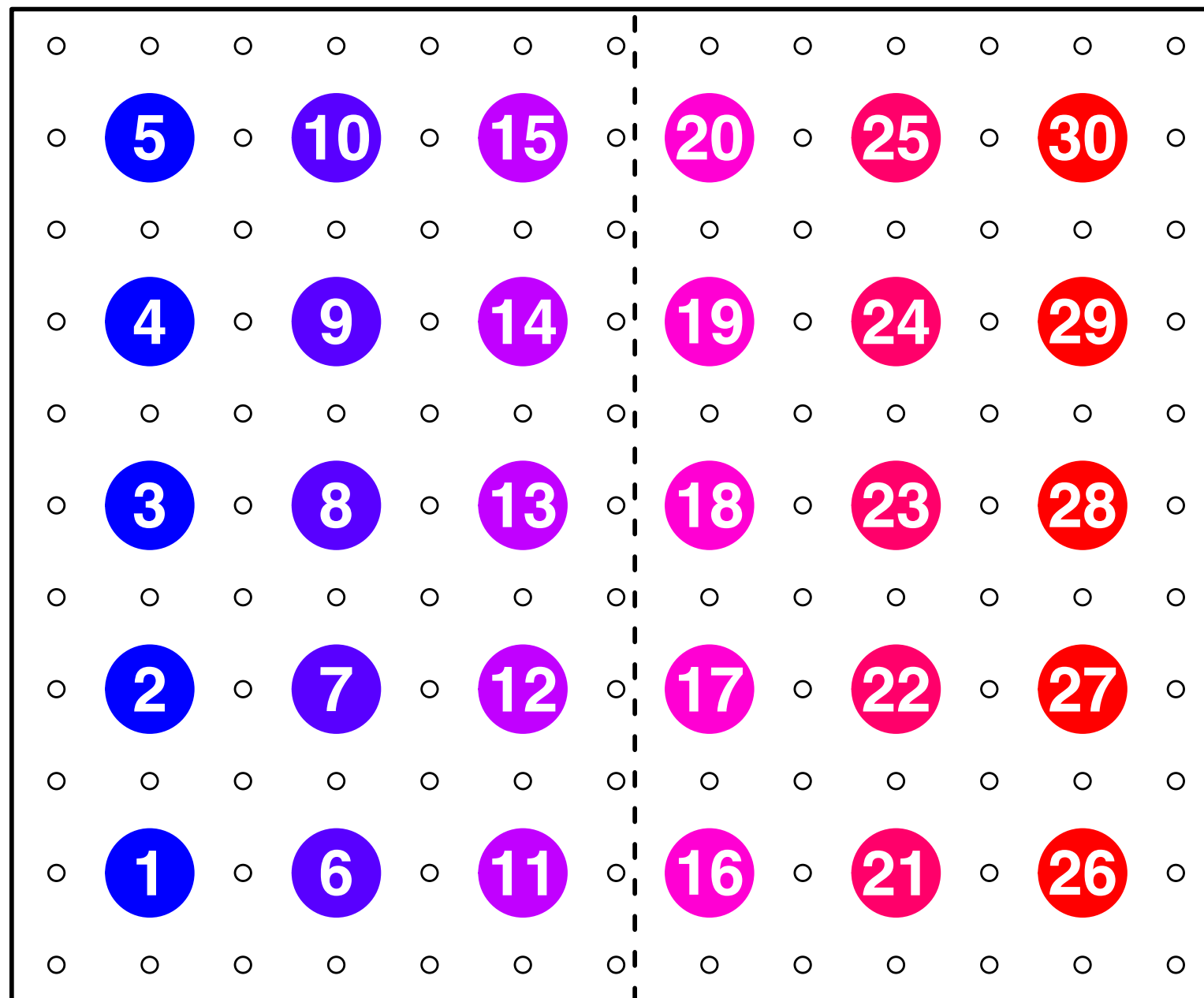
$$\Omega_{i,j} = \frac{1}{\alpha_0} \exp \left(-(\alpha_1 D_{i,j})^{\alpha_2} \right)$$

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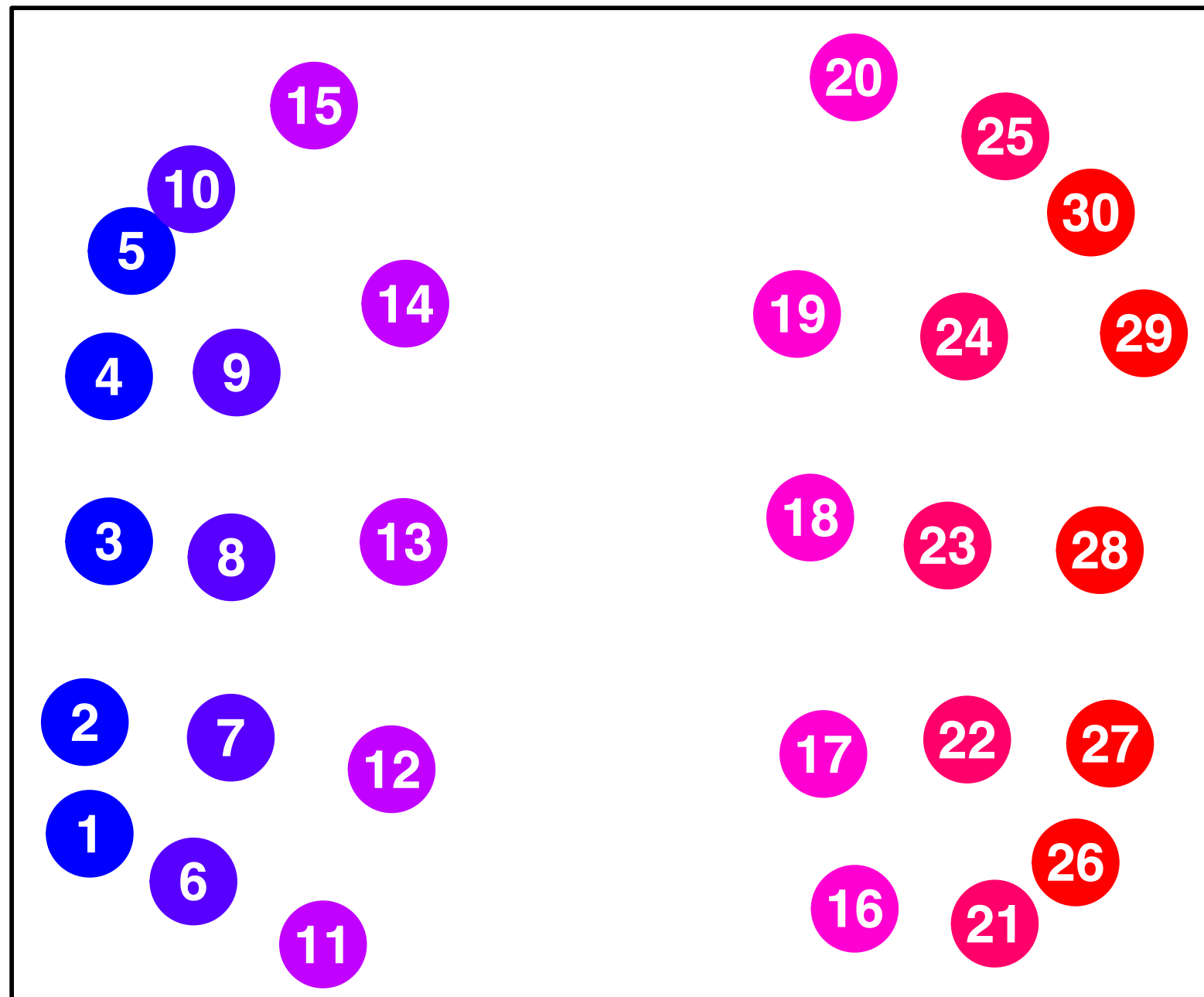
$$\Omega = \text{function}(\alpha_0, \alpha_1, \alpha_2, D(G'))$$

$$P(\Omega \mid \hat{\Omega}) \propto P(\hat{\Omega} \mid \Omega, L) P(\vec{\alpha}) P(G')$$

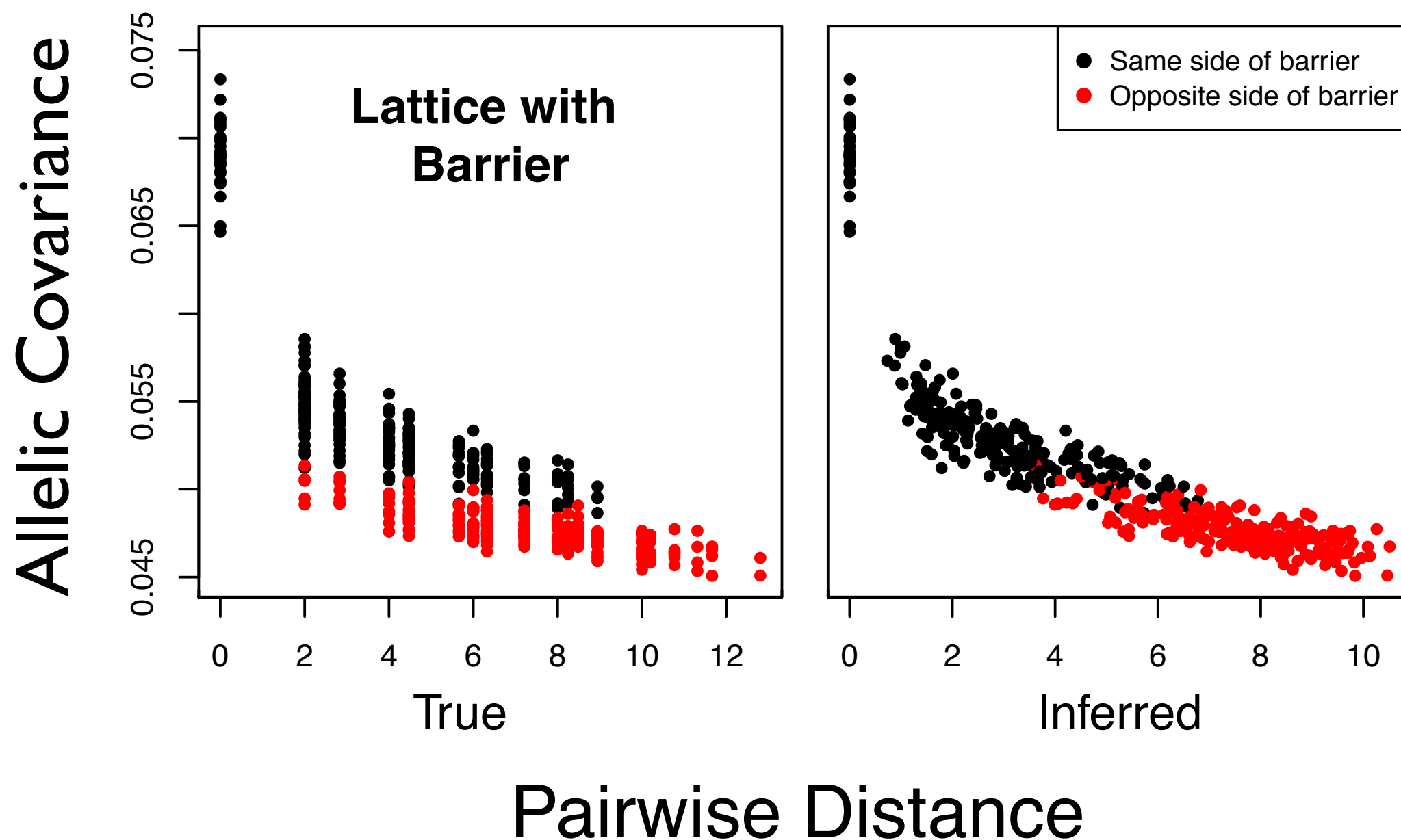
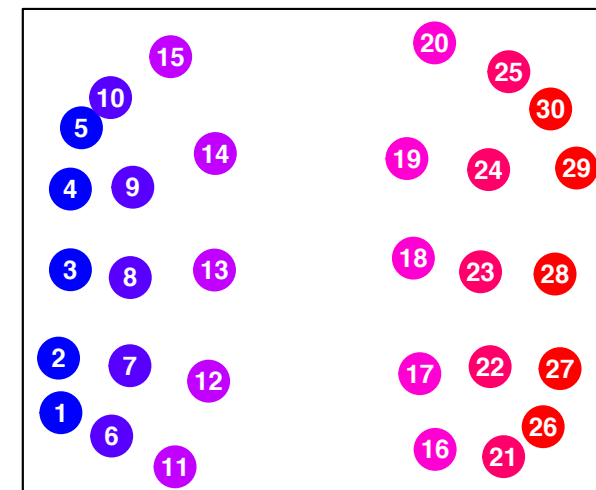
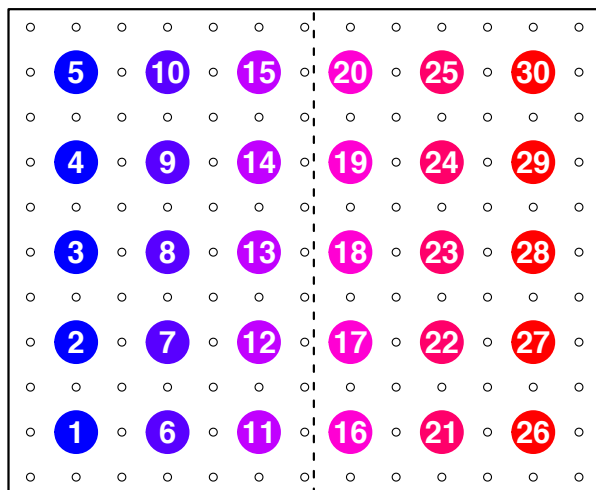
Take this out for a spin:
map with a gap



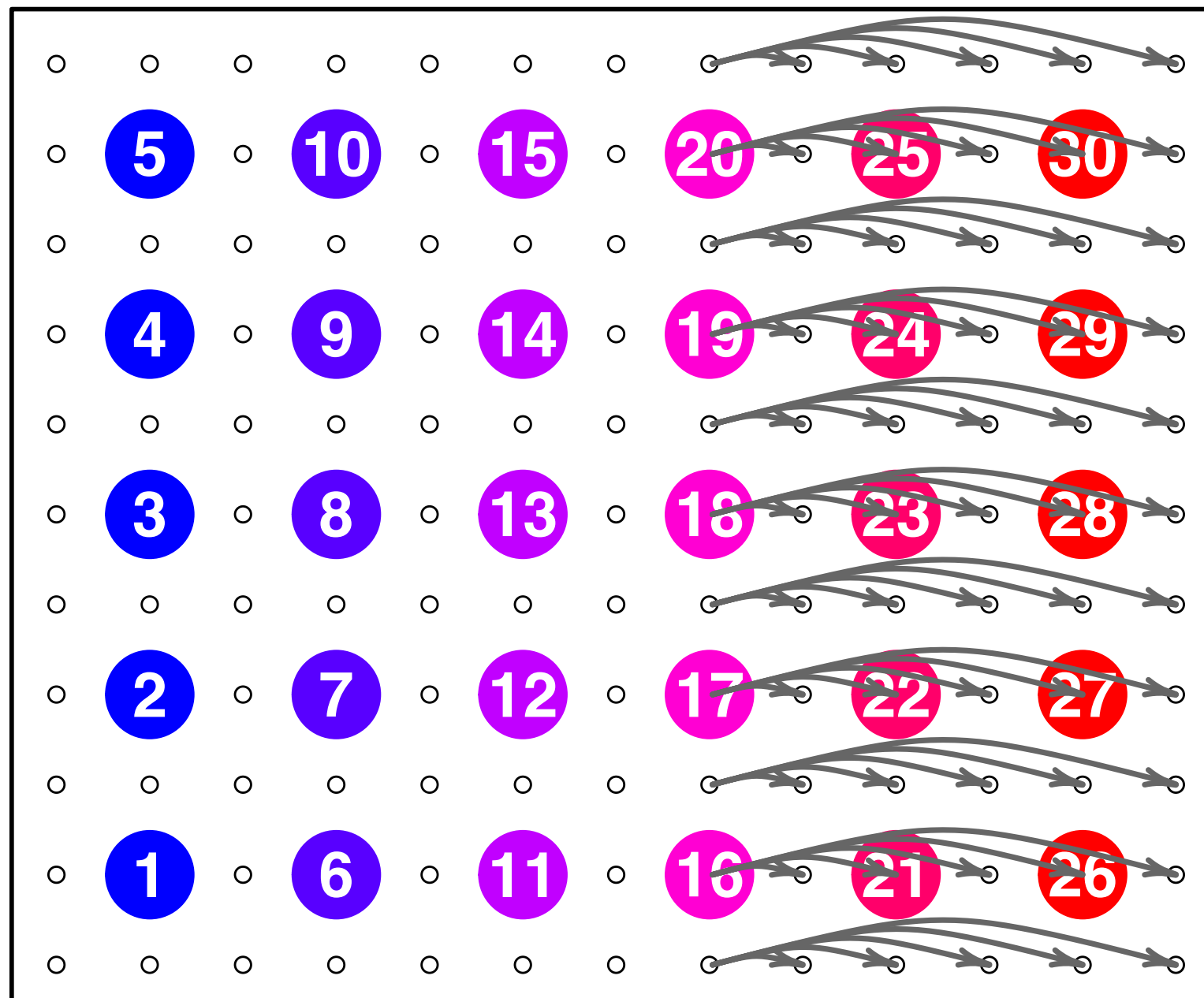
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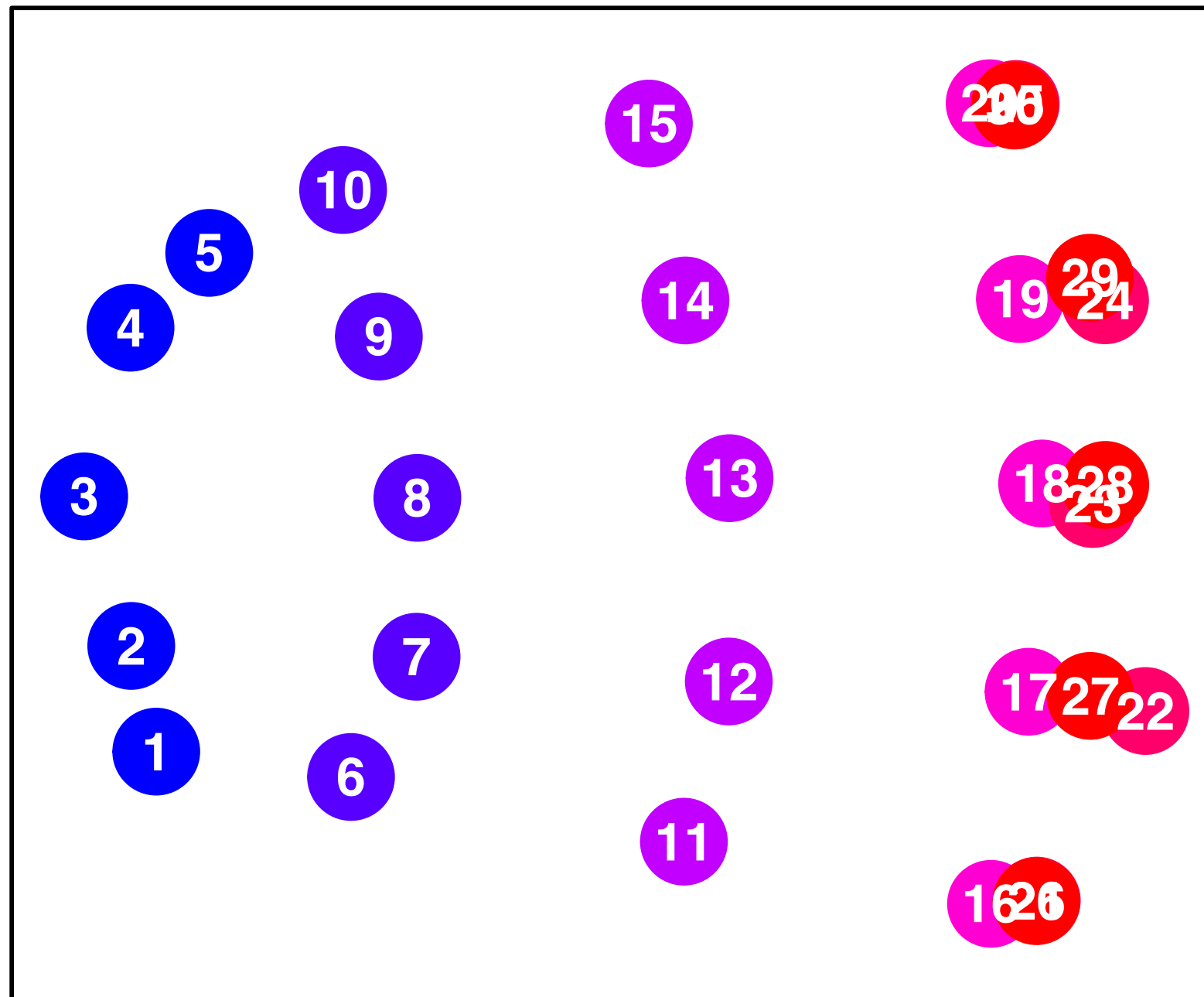
Take this out for a spin:
map with a gap



Take this out for a spin:
expansion event

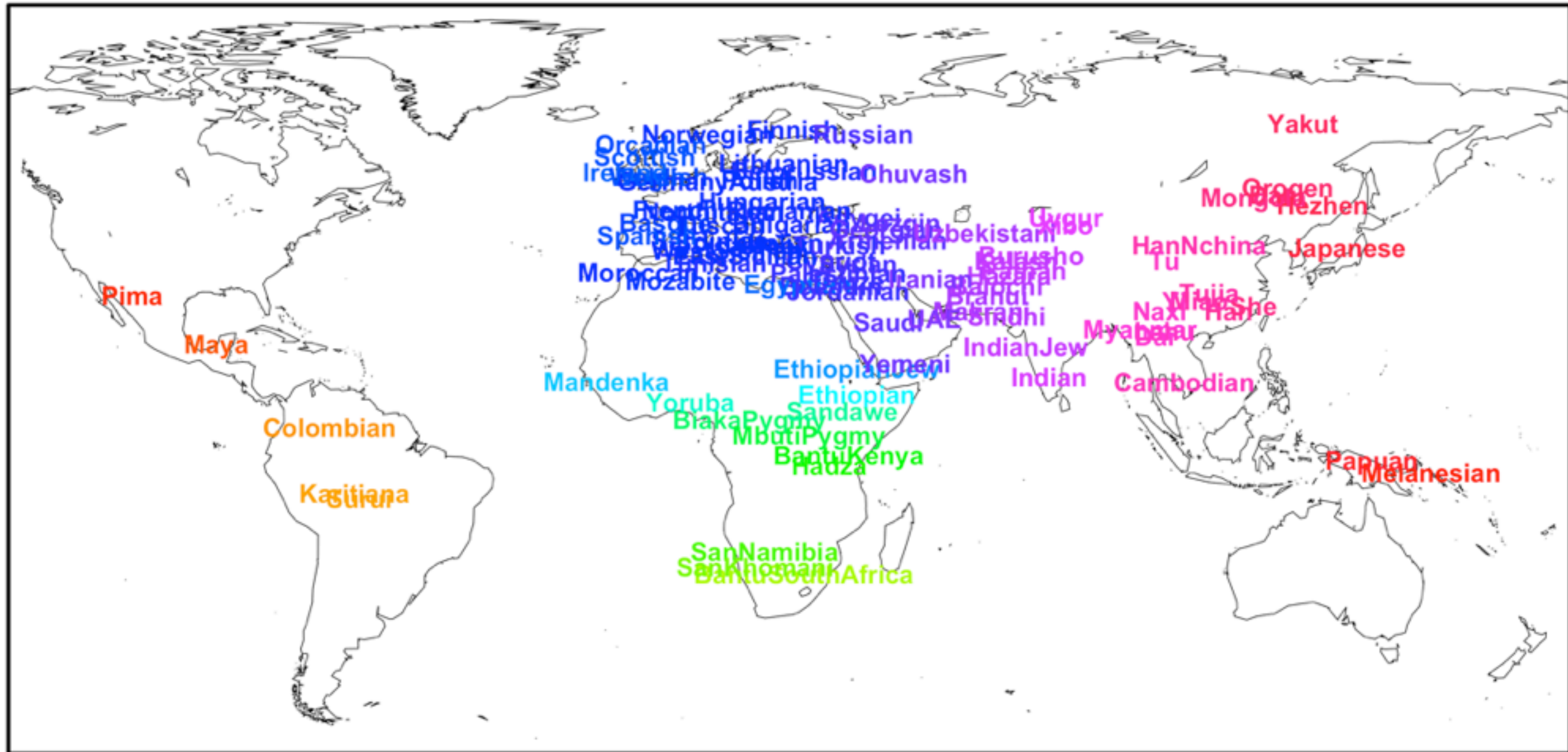


Take this out for a spin: expansion event



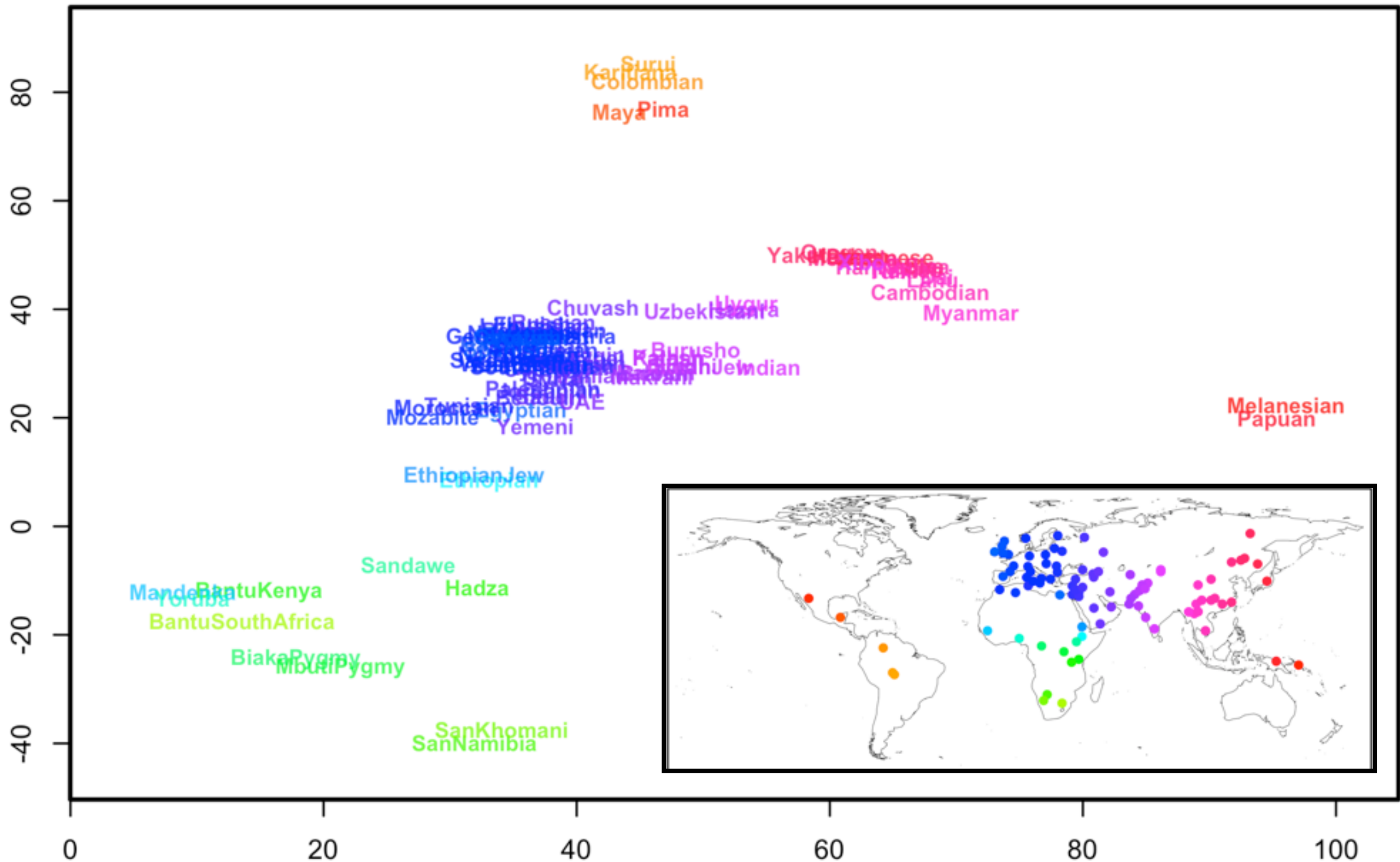
Empirical Application: Humans

Empirical Application: Humans

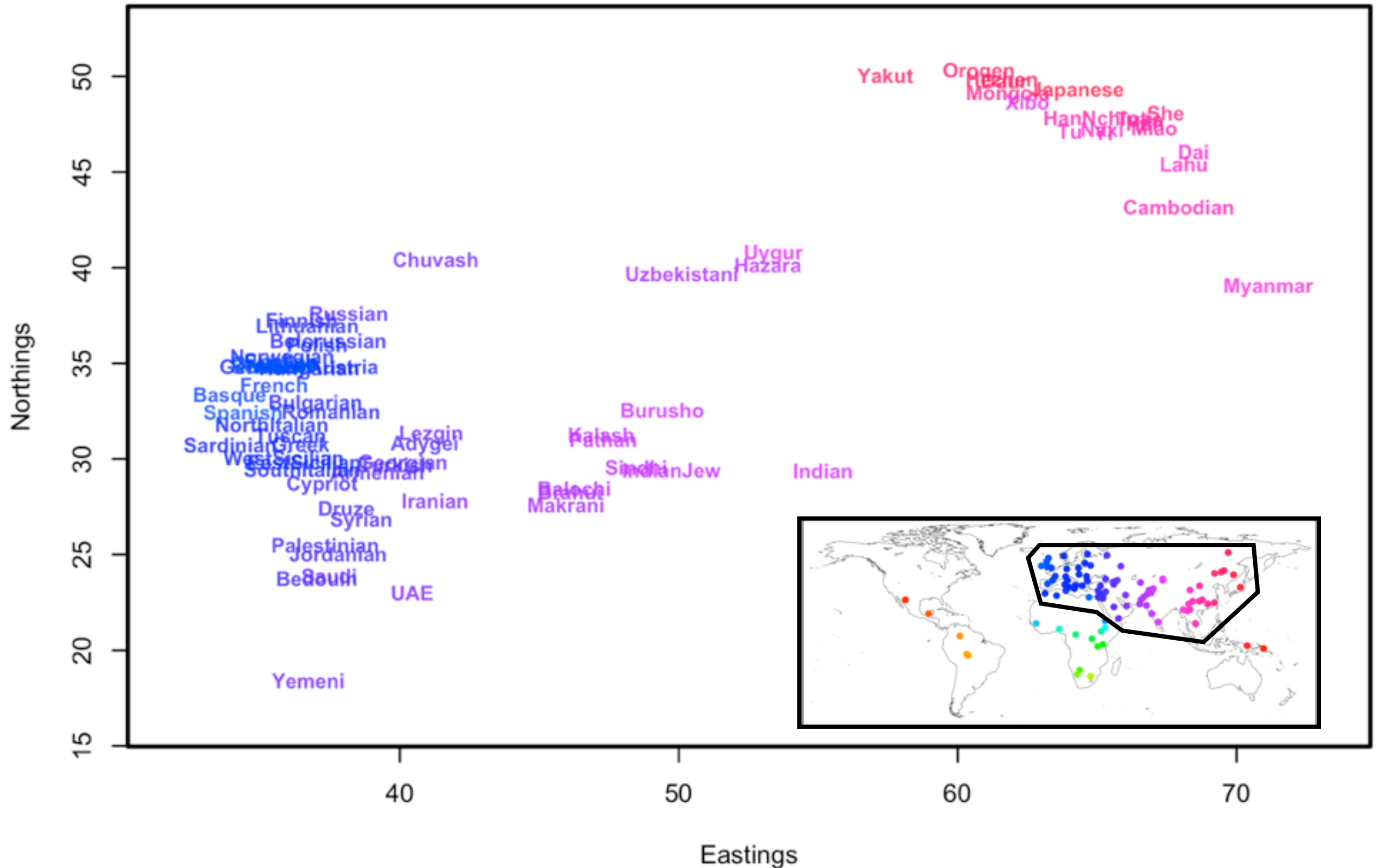


Hellenthal et al. 2014 Science

SpaceMix Map: Humans



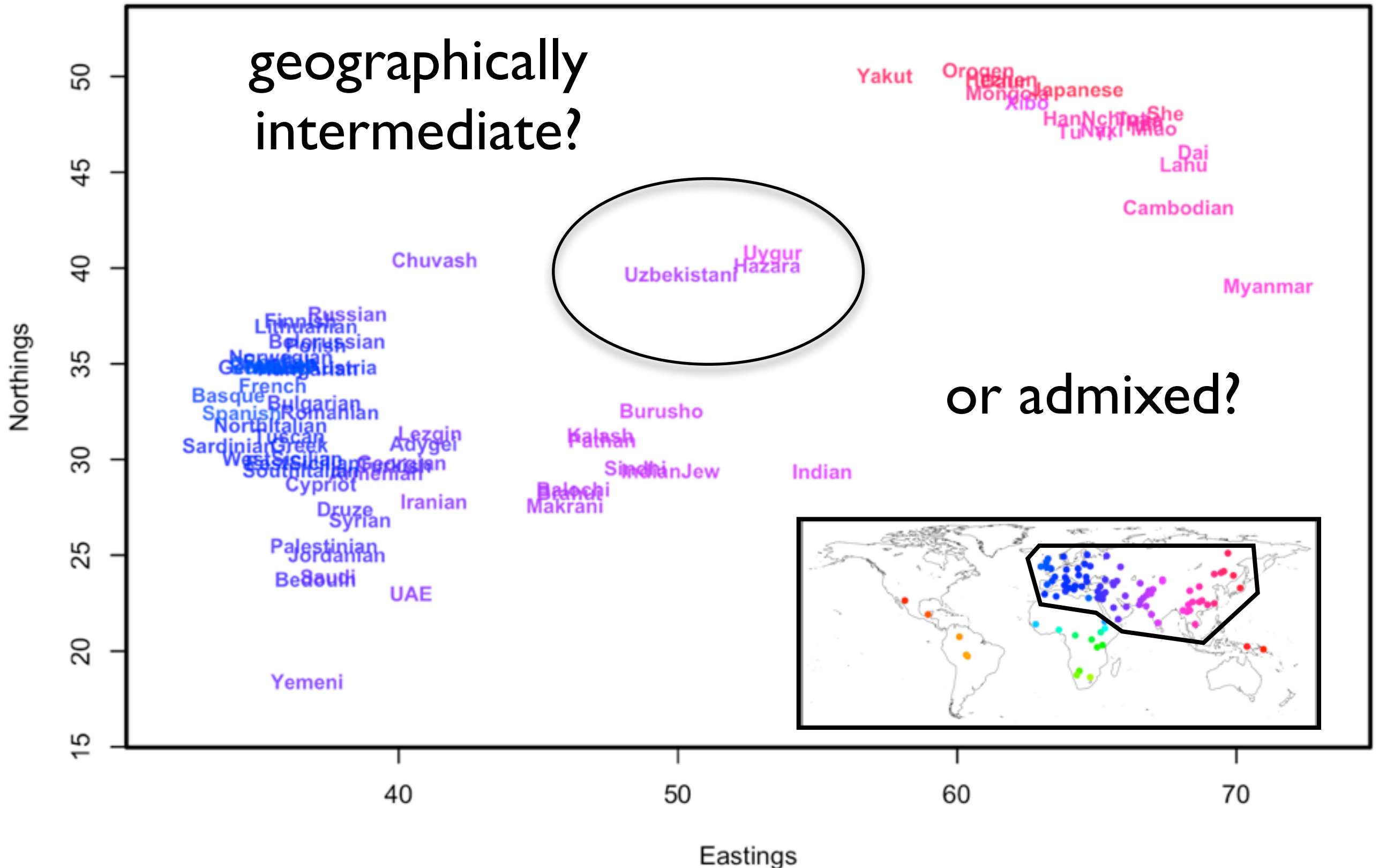
SpaceMix Map: Humans



SpaceMix Map: Humans

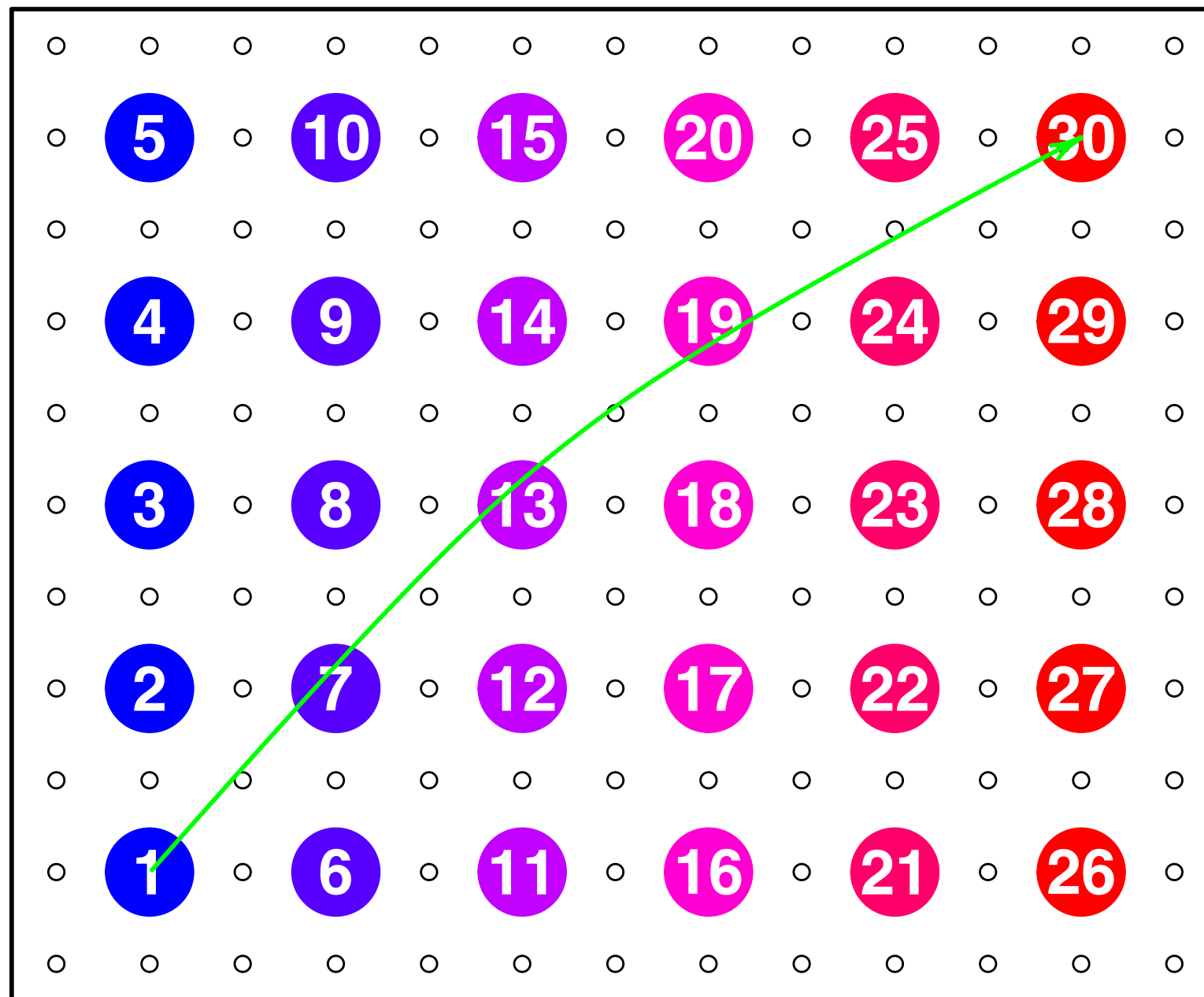
geographically
intermediate?

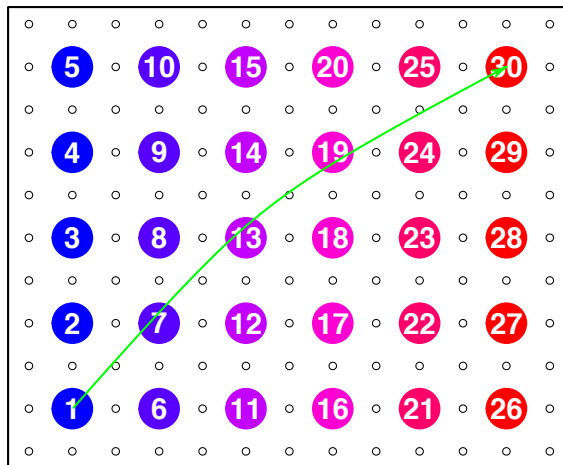
or admixed?



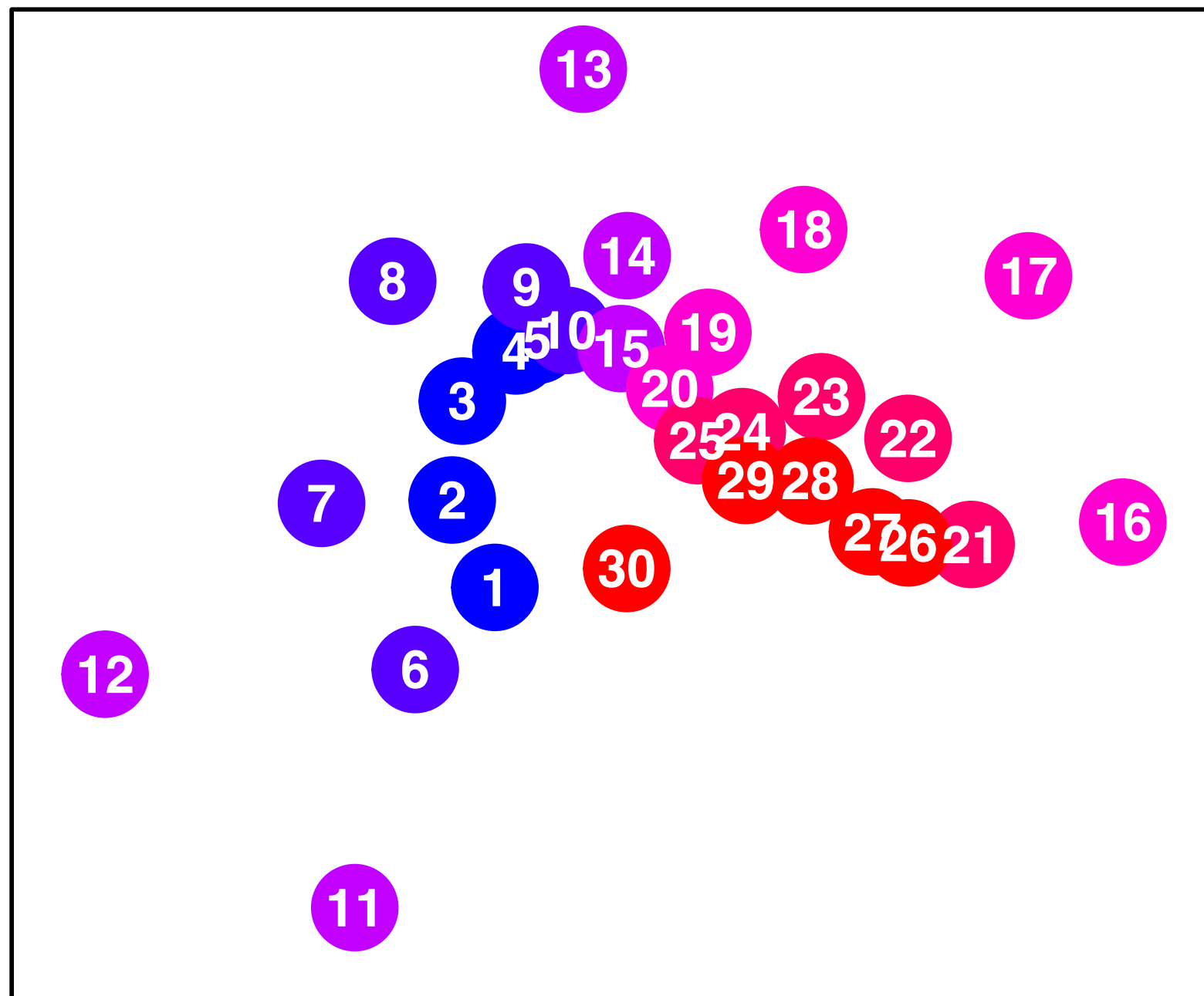
What happens if there's admixture?

What happens if there's admixture?

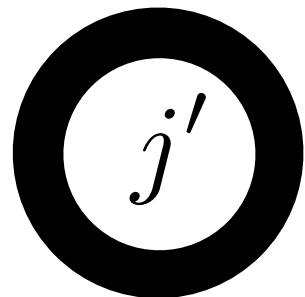




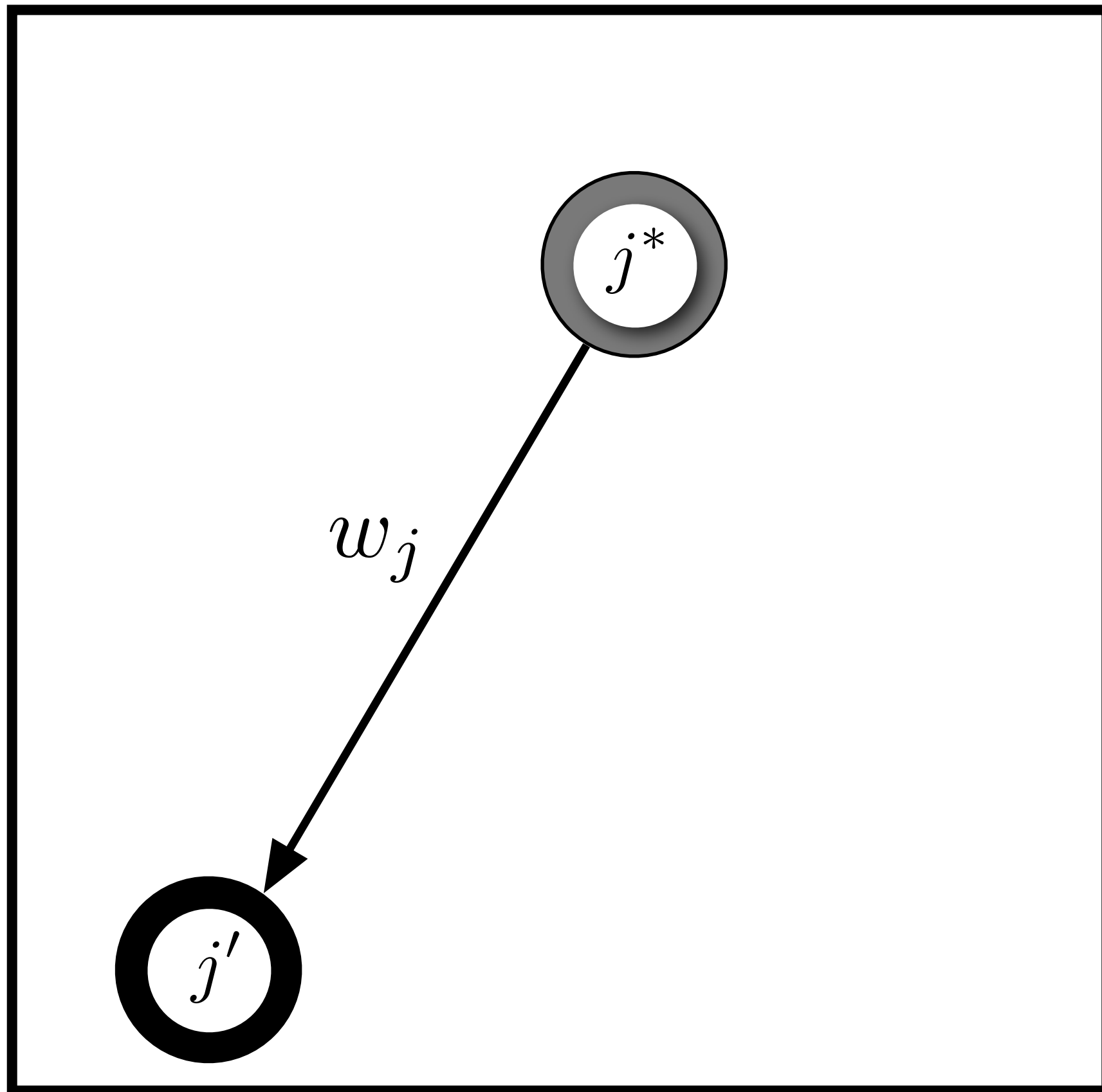
Admixture Simulation: corner admixture event



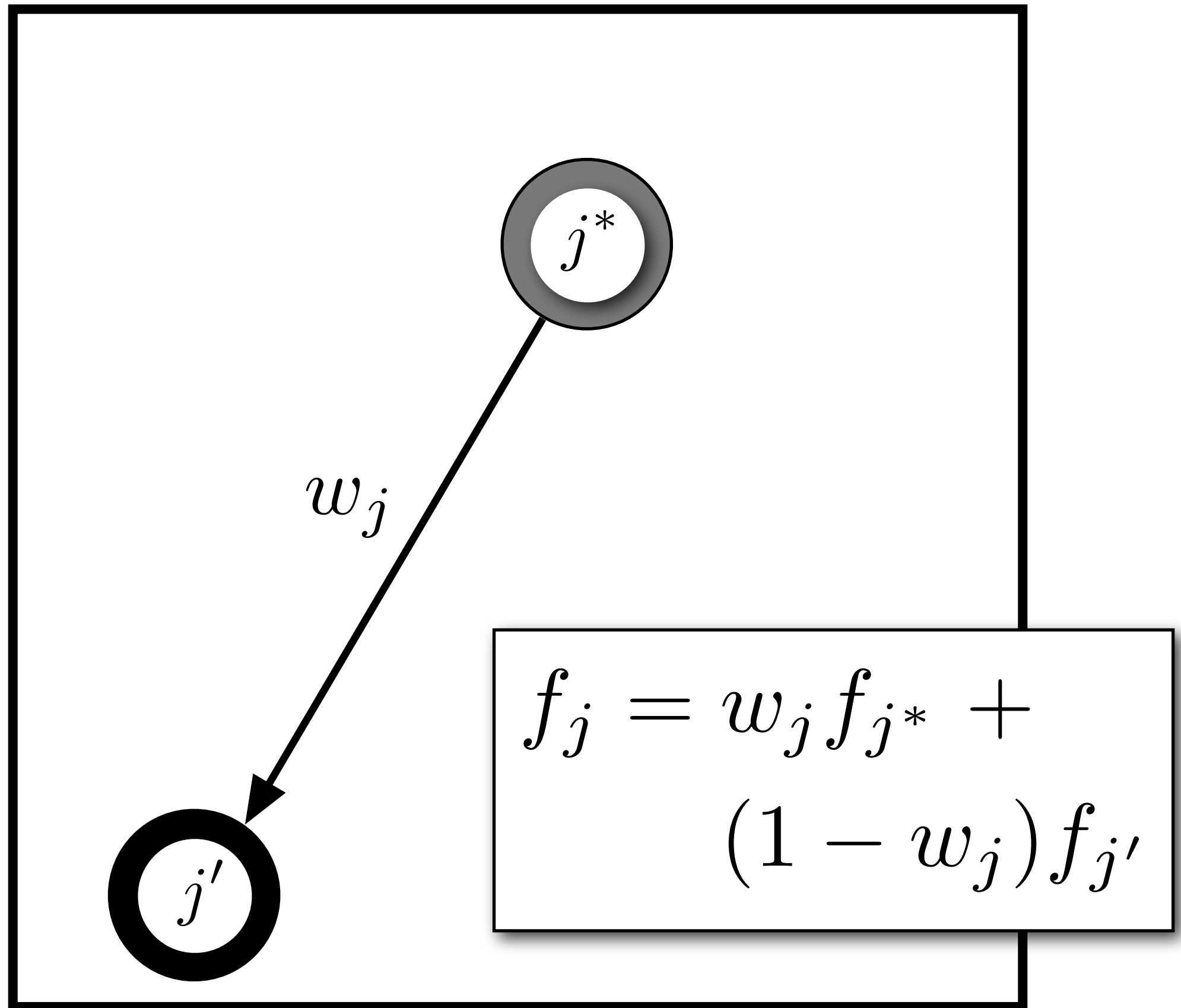
Explicitly modeling admixture



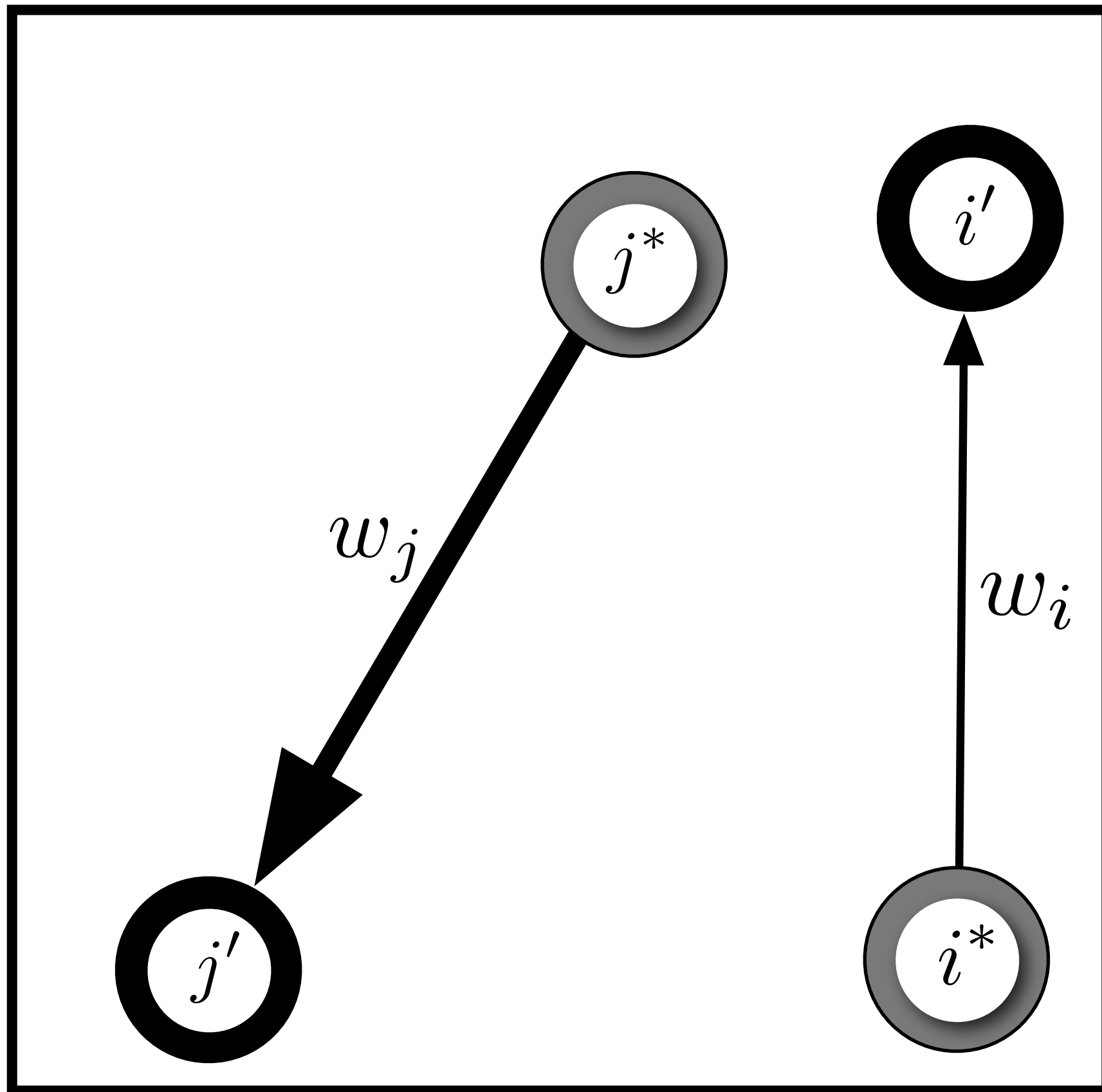
Explicitly modeling admixture



Explicitly modeling admixture



Explicitly modeling admixture



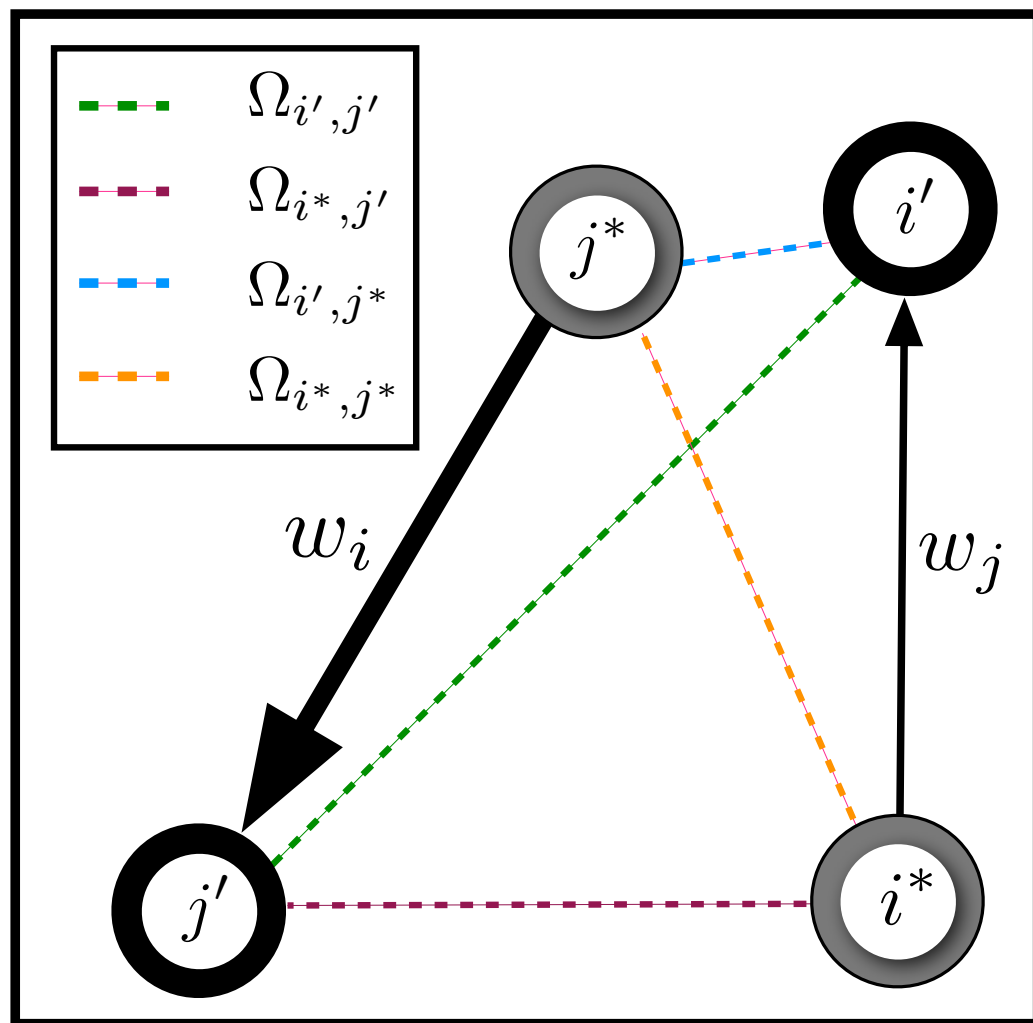
Explicitly modeling admixture

$$\Omega_{i,j}^* = (1 - w_i)(1 - w_j) \Omega(D_{i'}, j') +$$

$$w_i(1 - w_j) \Omega(D_{i^*}, j') +$$

$$w_j(1 - w_i) \Omega(D_{i'}, j^*) +$$

$$w_i w_j \Omega(D_{i^*}, j^*)$$



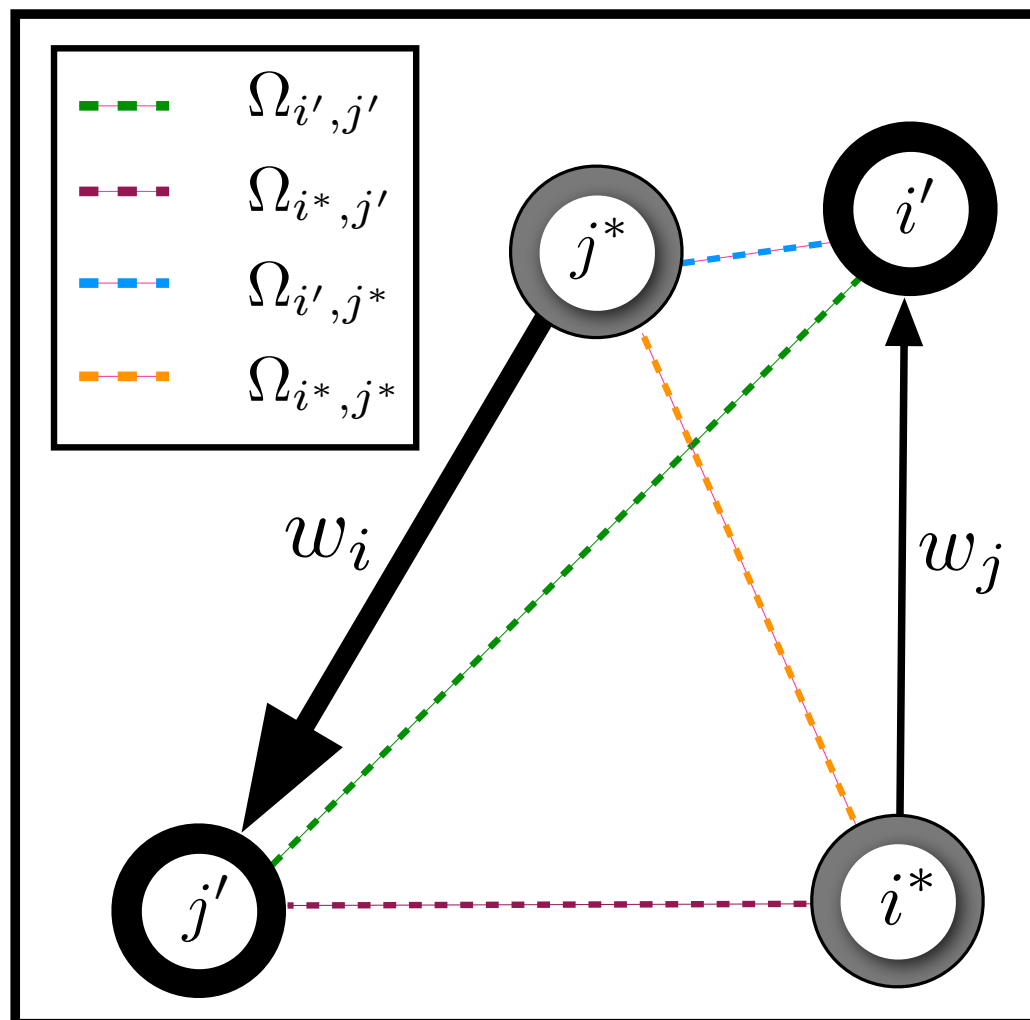
Explicitly modeling admixture

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$$P(\Omega^* \mid \hat{\Omega}) \propto P(\hat{\Omega} \mid \Omega^*, L) P(\vec{\alpha})$$

$$P(G')$$

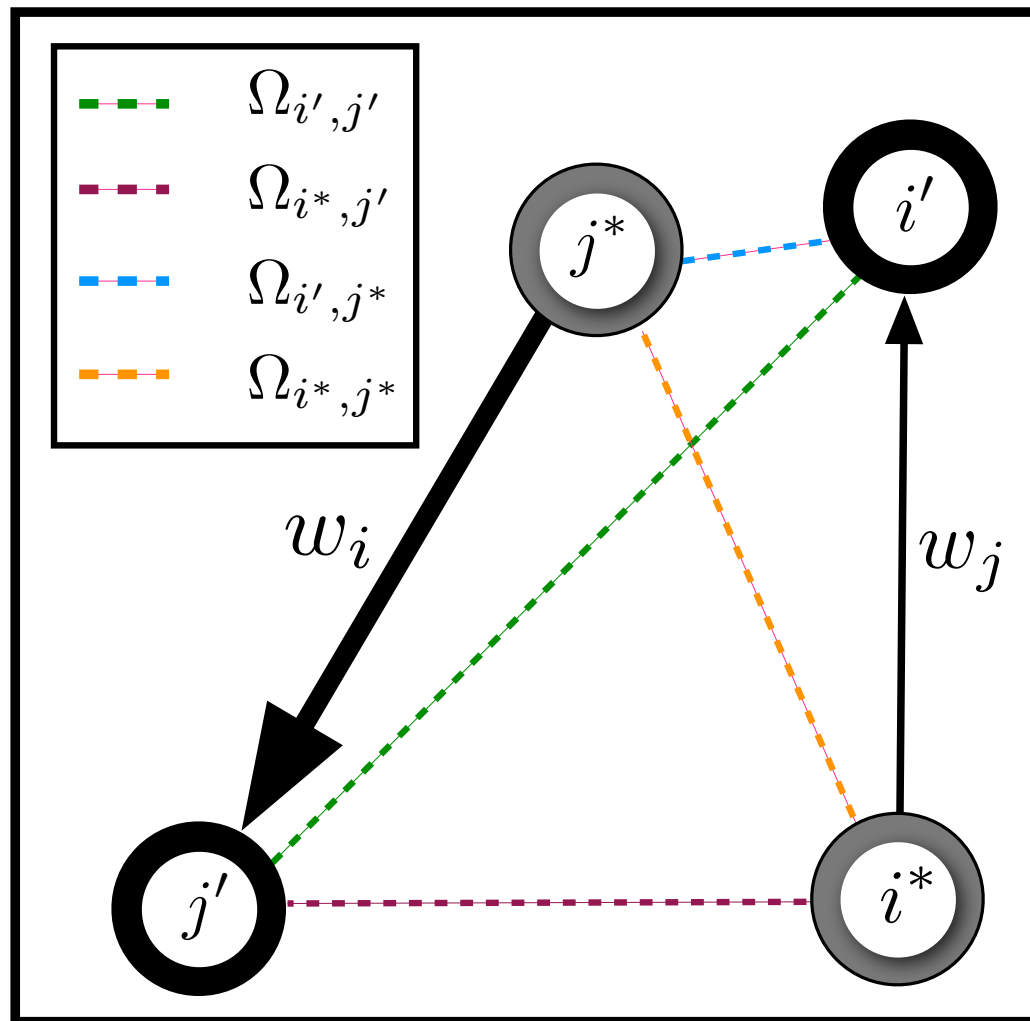
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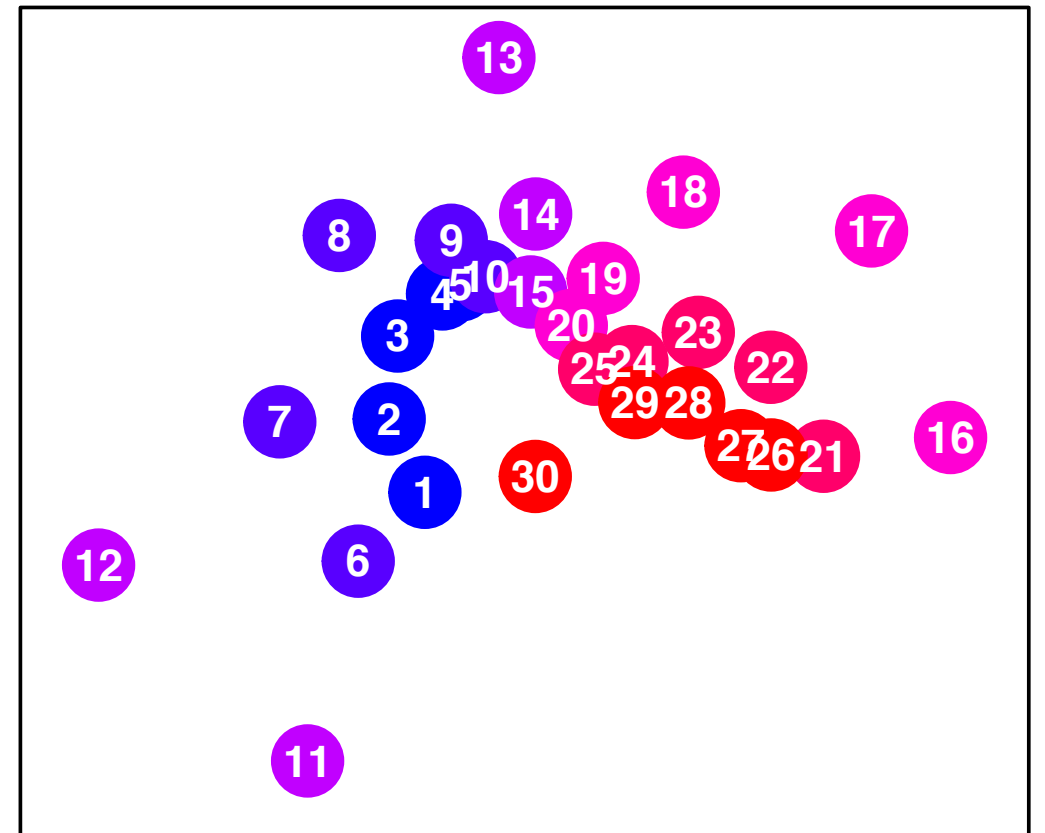
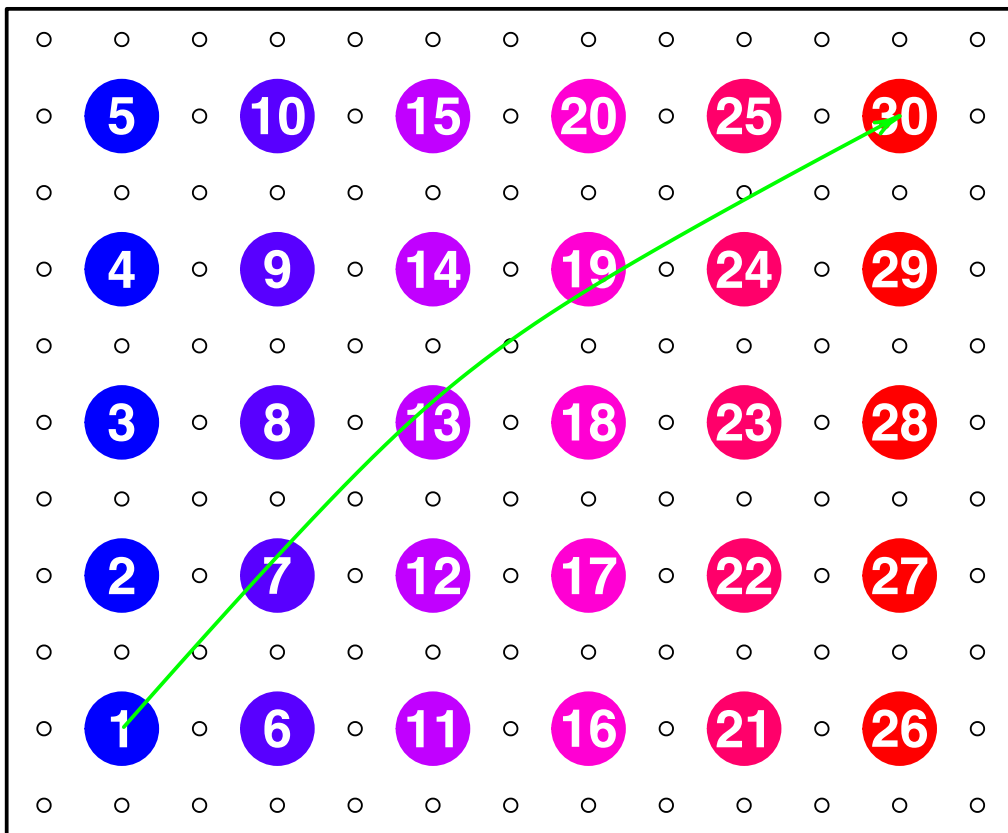
$$w_i w_j \Omega(D_{i^*}, j^*)$$



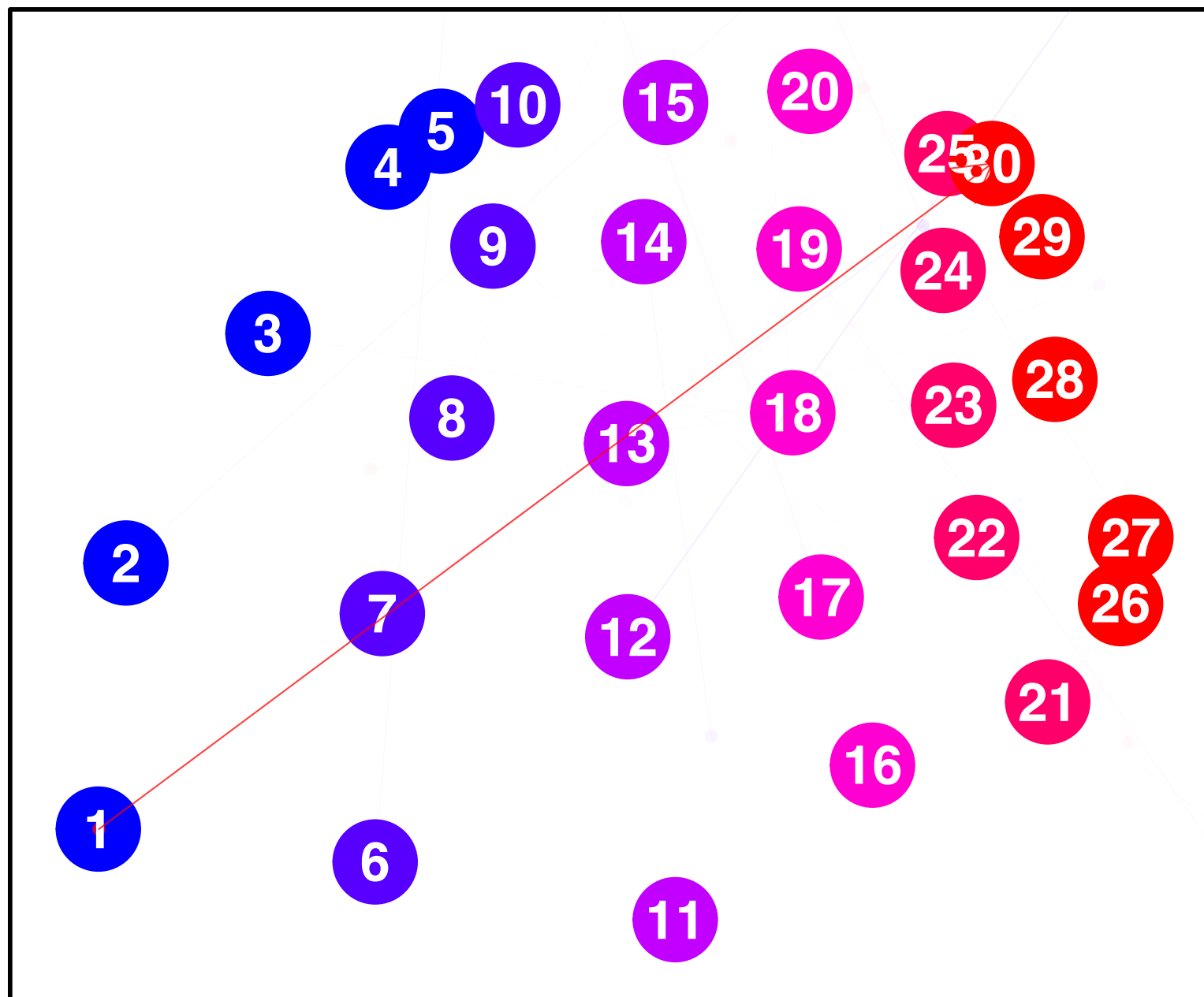
$$P(\Omega^* \mid \hat{\Omega}) \propto P(\hat{\Omega} \mid \Omega^*, L) P(\vec{\alpha})$$

$$P(G') P(G^*) P(w)$$

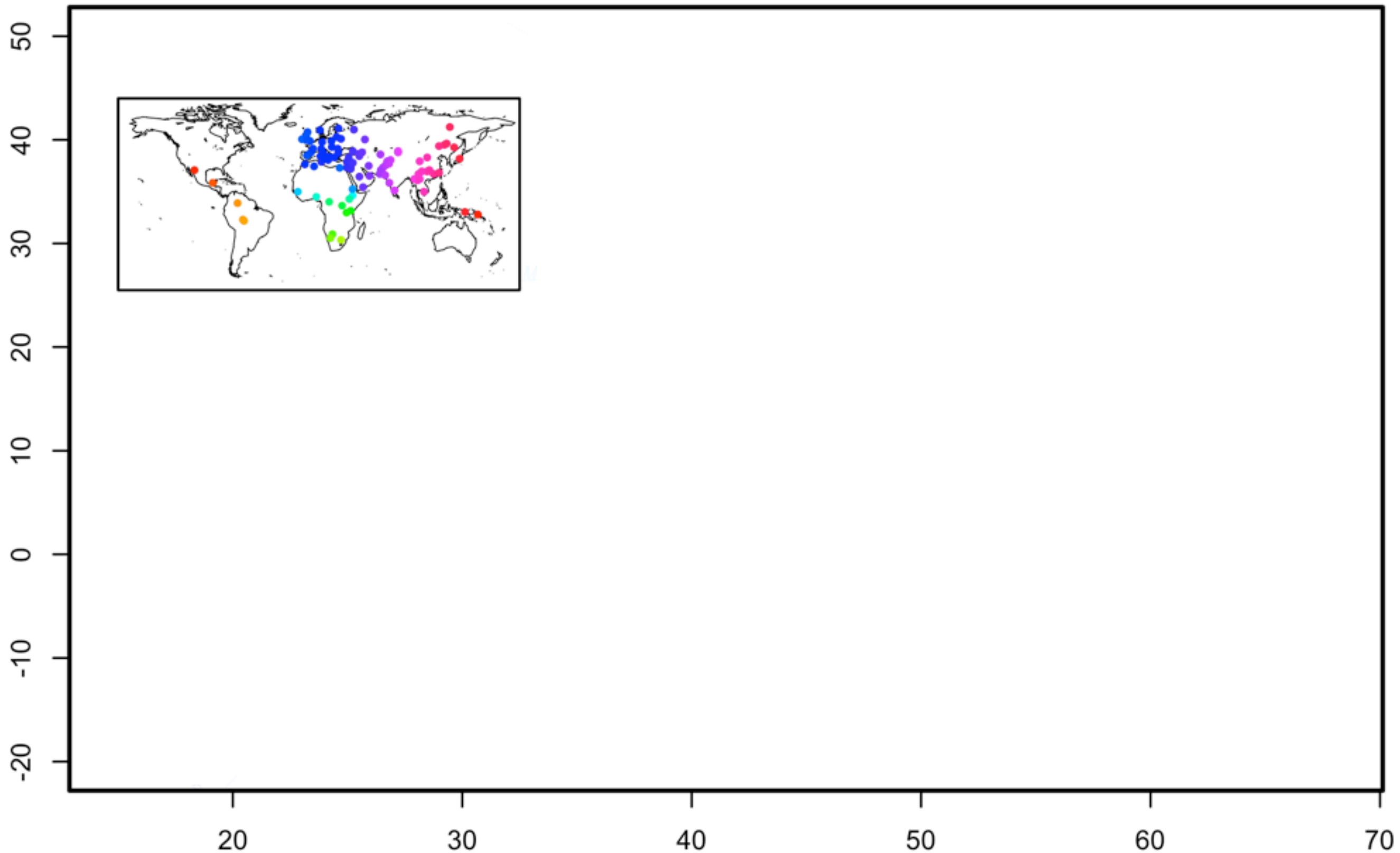
Admixture Simulation: corner admixture event



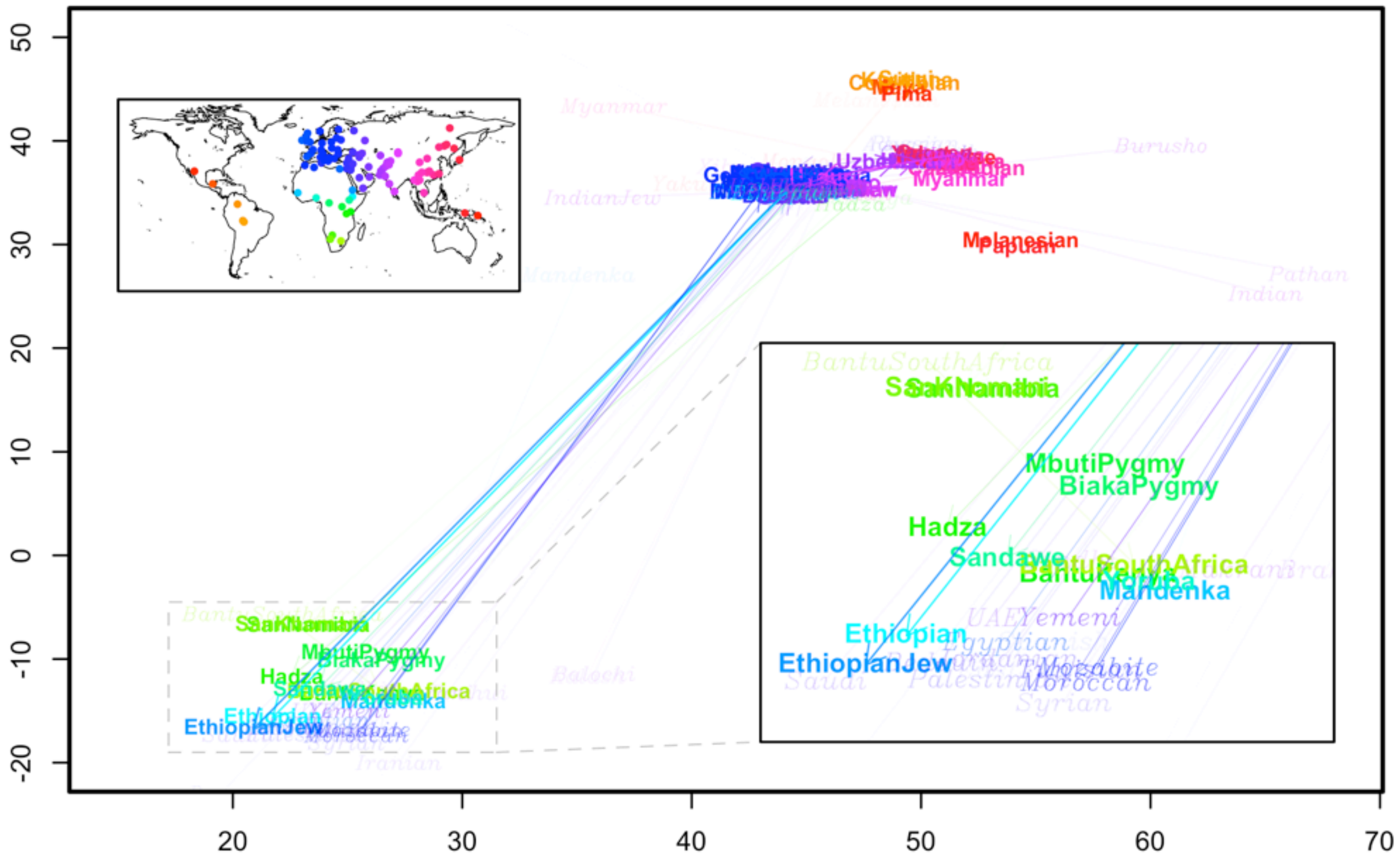
Modeling admixture: corner admixture event



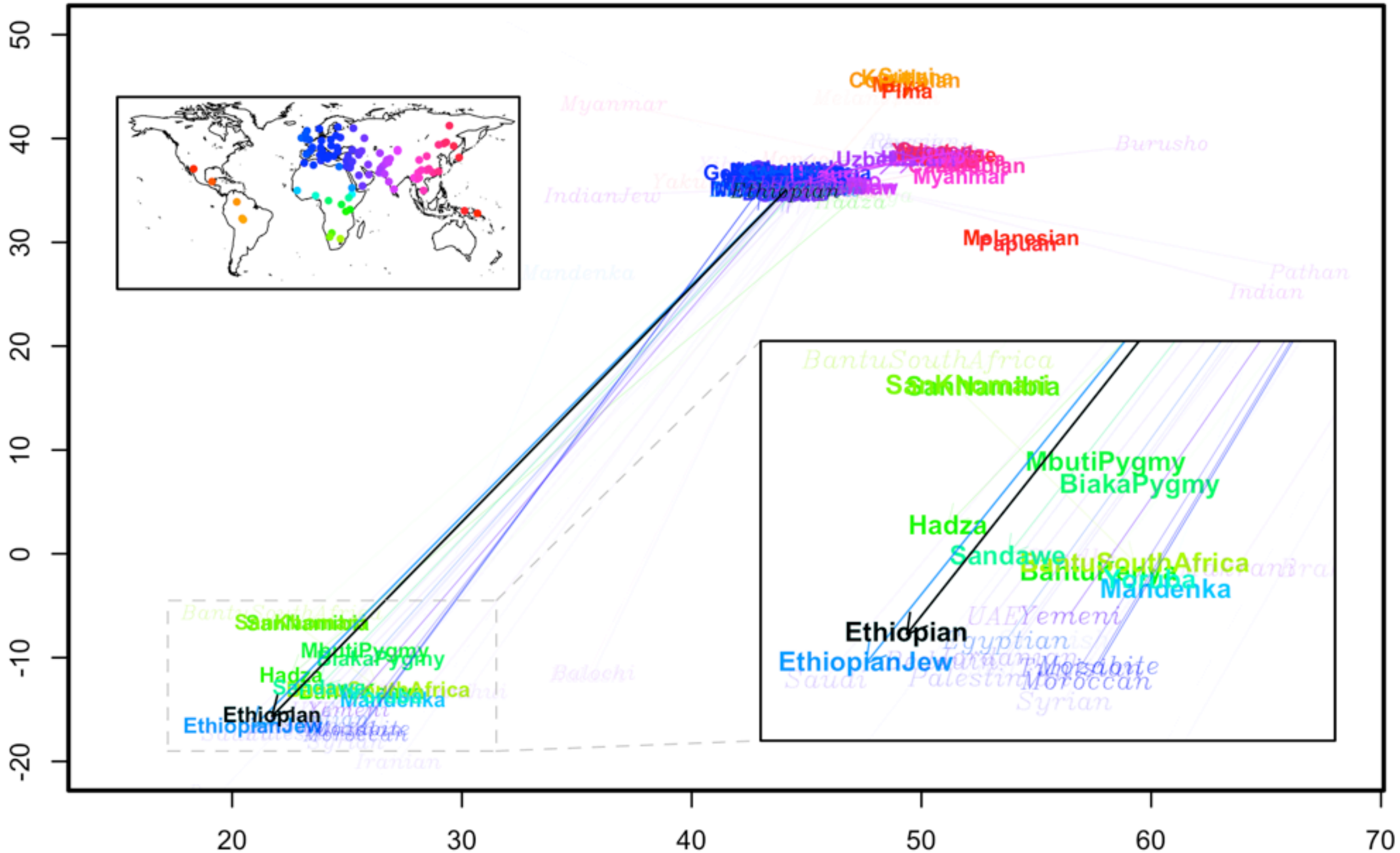
SpaceMix Map: Humans with Admixture



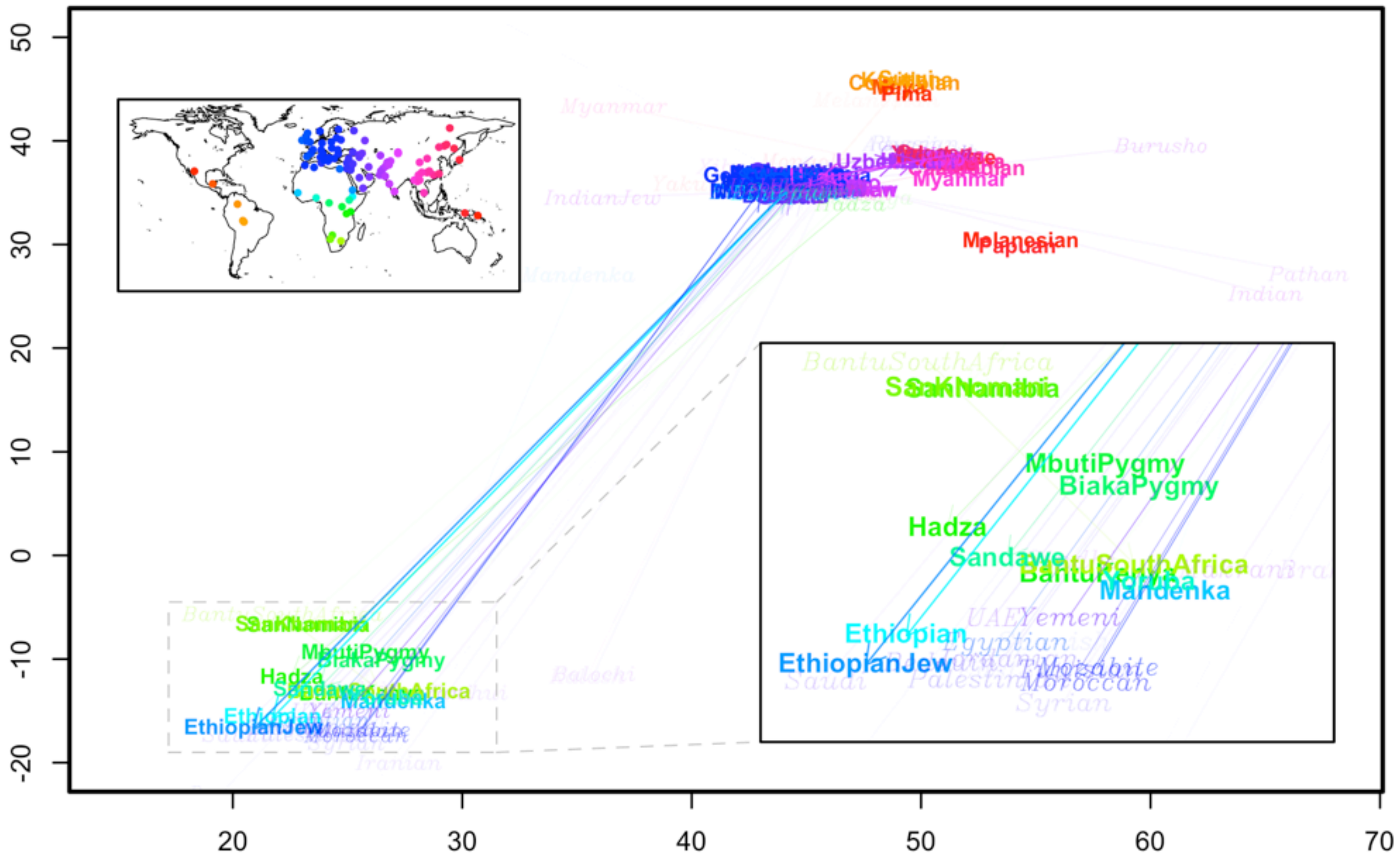
SpaceMix Map: Humans with Admixture

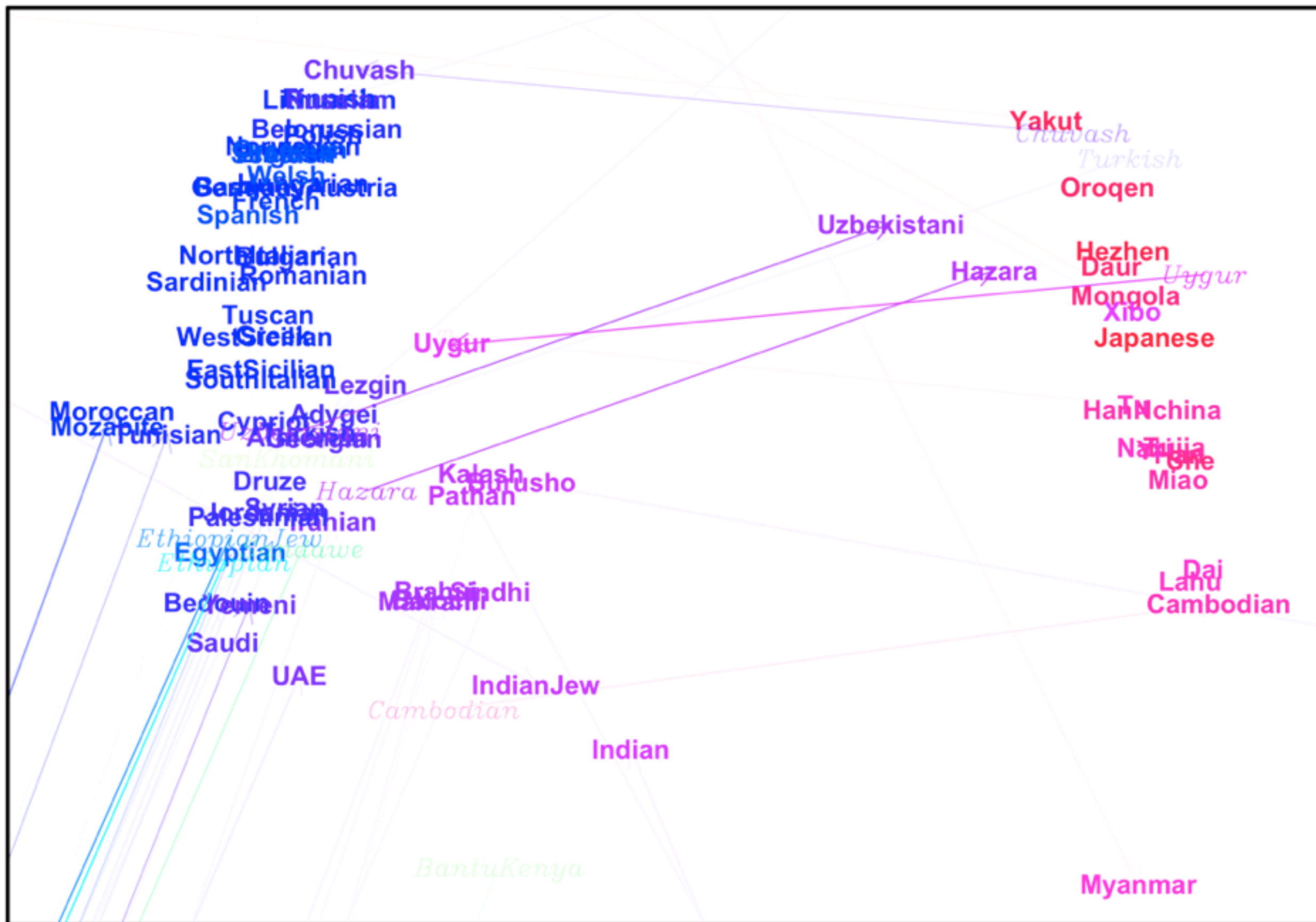


SpaceMix Map: Humans with Admixture



SpaceMix Map: Humans with Admixture





SpaceMix

Conclusions

Extensions

Collaborators:

Graham Coop
Peter Ralph

Funding:

NSF, CPB

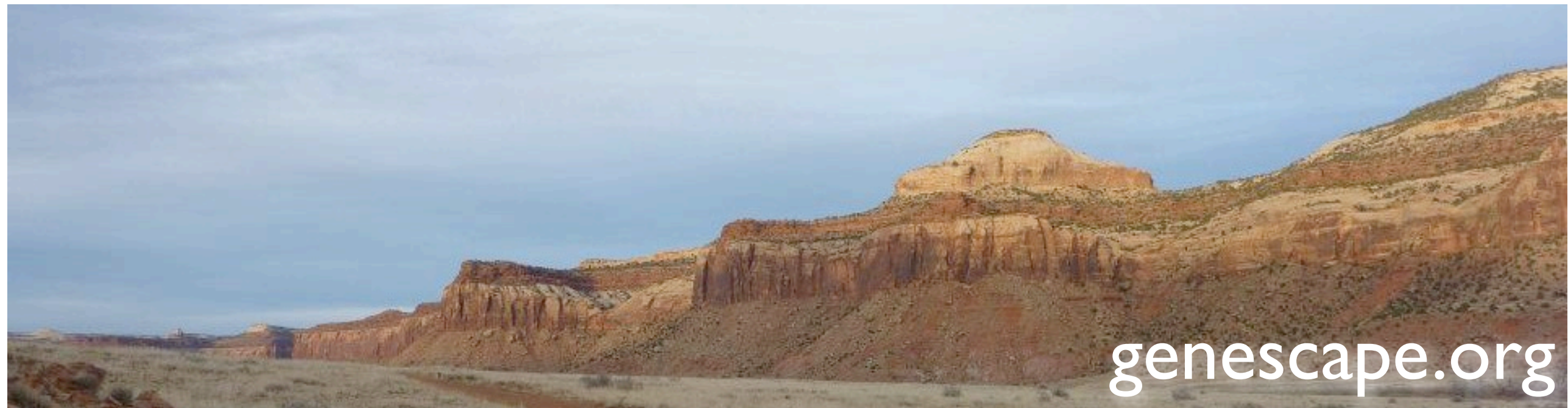
Thanks To:

Wisdom:

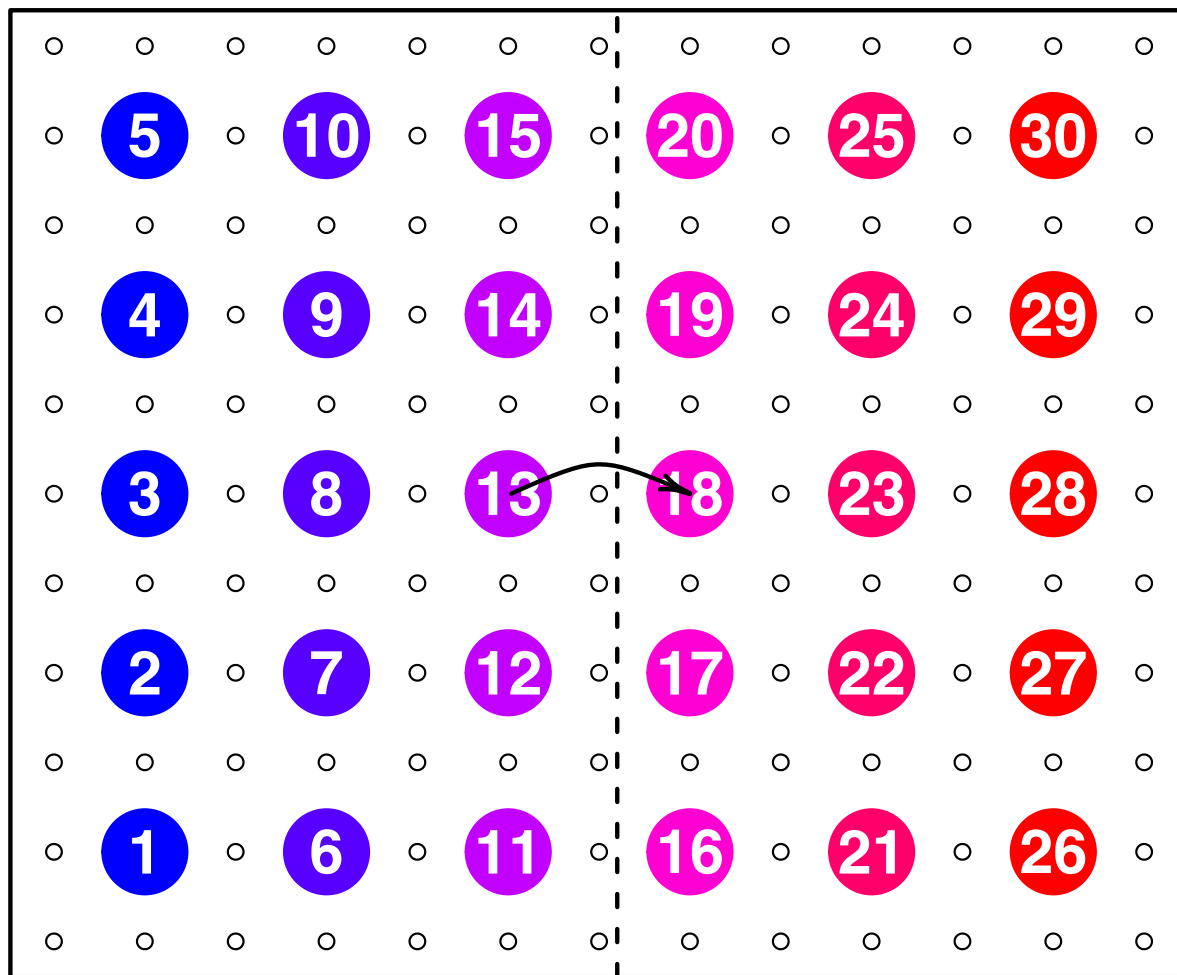
Yaniv Brandvain
Marjorie Weber
Luke Mahler
Will Wetzel
Coop Lab
Brad Shaffer

Data:

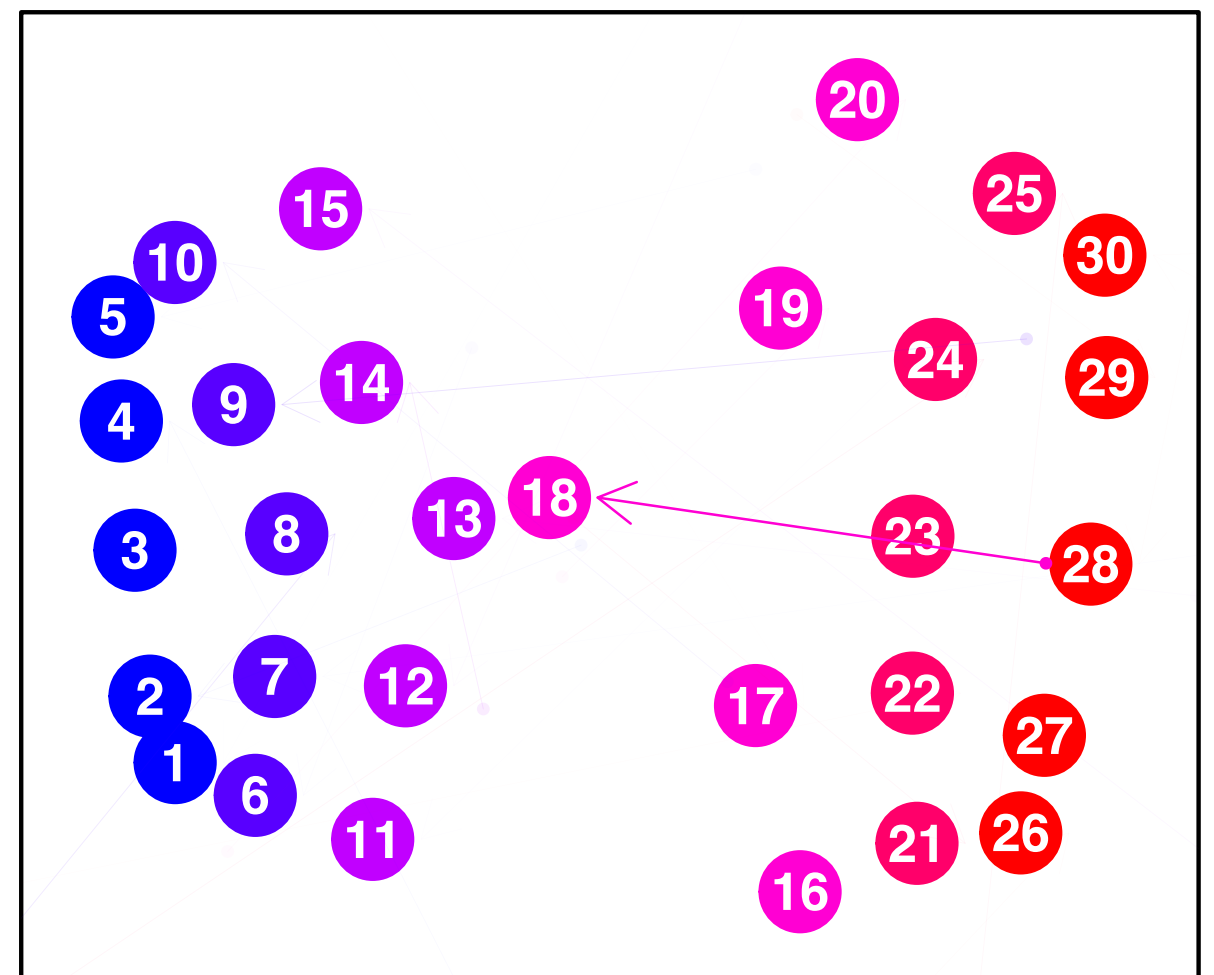
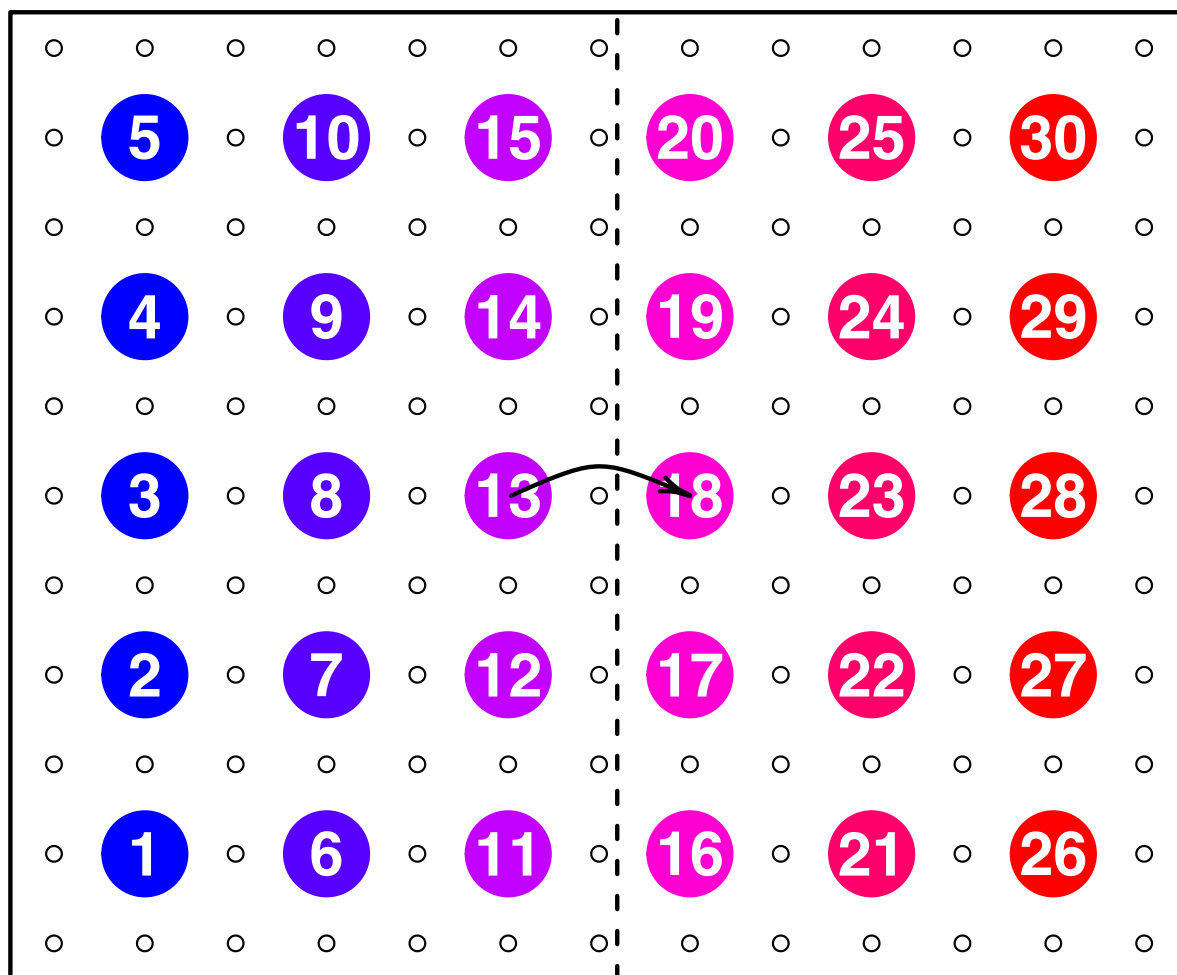
Garrett Hellenthal



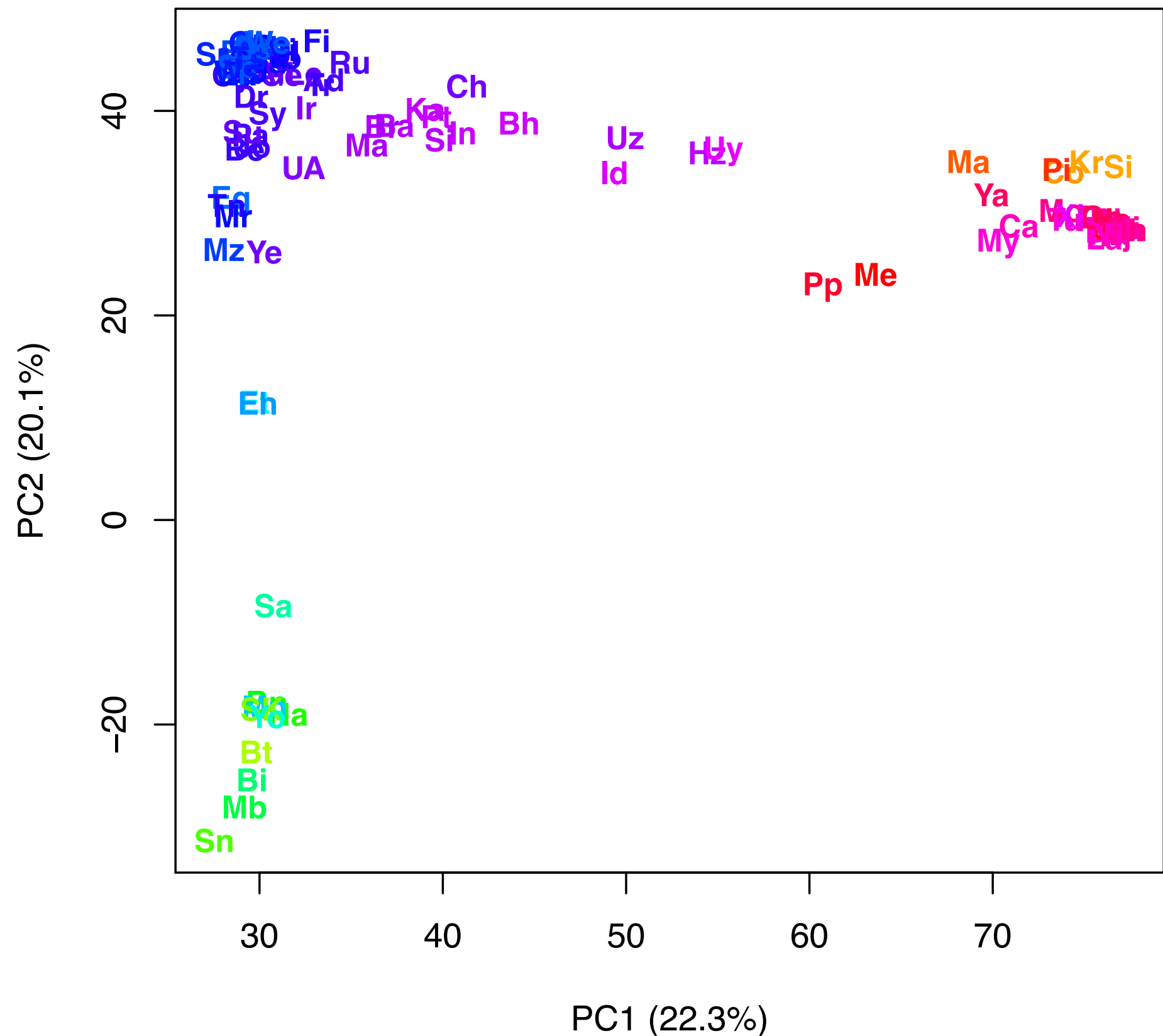
Modeling admixture: corner admixture event



Modeling admixture: corner admixture event



PCA map of Human samples



Human Population Admixture Proportions

